* Discrete Fourier Transformation -

→ Transforms N input Samples $x(n)$ into N Complex frequency Samples $X(K)$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{2\pi K n}{N} j}$$

Spatial domain → frequency domain

Inverse DFT

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi K n}{N}}$$

→ used in filter design, Convolution, Spectral ~~Anay~~ Analysis

→ DFT takes $O(N^2)$ while FFT takes $O(N \log N)$

Exa- Compute DFT of the seq.

$$x(n) = \{1, 0, 0, 1\}$$

$N = 4$

$$X(K) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi K n}{N}}$$

$$= x(0)e^{0} + x(1)\, e^{-j\frac{2\pi K}{4}} + x(2)\, e^{-j2\pi K} + x(3)\, e^{-j\frac{3\pi K}{2}}$$

$$X(K) = 1 + e^{-j\frac{3\pi K}{2}}$$

$K = 0, 1, \ldots, N-1$

$X(0) = 1 + e^{0} = 2$

$X(1) = 1 + e^{-j3\pi/2} = 1 + j$

$X(2) = 1 + e^{-j3\pi} = 1 - 1 = 0$

$X(3) = 1 + e^{-j9\pi/2} = 1 - j$

$$X(K) = \{2, 1+j, 0, 1-j\}$$

⇒ Kernel of 4 Point DFT

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \rightarrow \text{Kernel}$$

1D DFT of image $f(x)$:

$$X(K) = \text{Kernel} * f(x)$$

2D DFT of image ~~f(x)~~ $f(x,y)$:

$$X(K,l) = \text{Kernel} * f(x,y) * \text{Kernel}^T$$

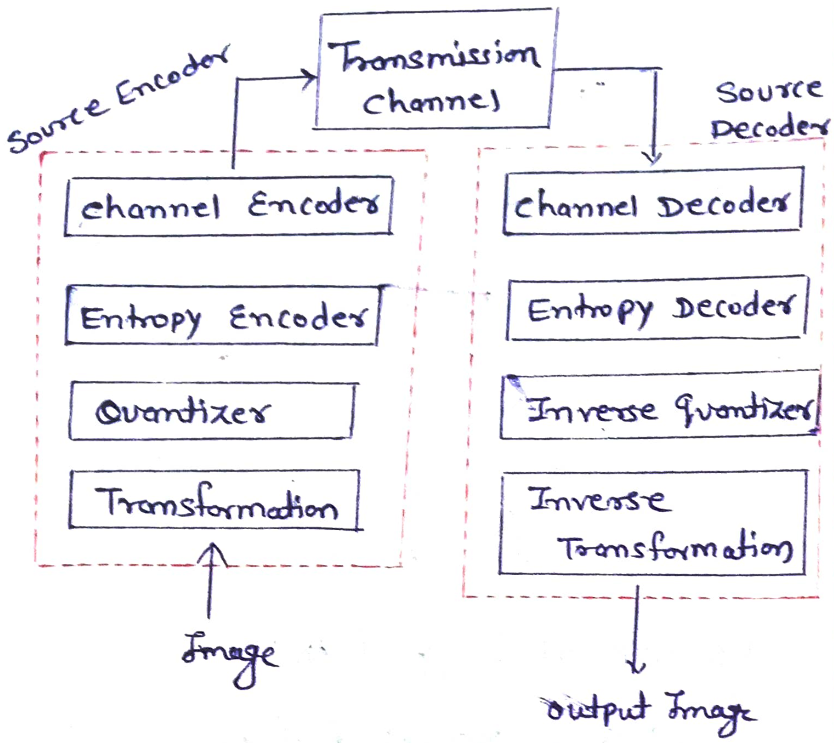
* Discrete Cosine Transformation

→ Converts spatial pixel intensity into a sum of Cosine fn, oscillating at different frequencies.

→ Energy Compaction:

DCT Packs the most crucial visual info. (Low freq. Component) into the top left corner of an image, while high freq. Noise is moved to the right bottom Corner.

Packs most signal energy into a few Co-efficients ⇒ Lossy Compressio



- **1D- DCT**

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{(2x+1)\pi u}{2N}\right]$$

Where $0 \leq K \leq N-1$

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}}, & \text{if } u=0 \\ \sqrt{\frac{2}{N}}, & \text{if } u \neq 0 \end{cases}$$

- **2D DCT :-**

$$C(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)\pi u}{2N}\right] \cos\left[\frac{(2y+1)\pi v}{2N}\right]$$