

* BNF Notations - (Backus Naur form)

- Notation Technique for CFGs
- Specify the syntax of the Language.

name ::= expansion
└─ may expand into

Every name in BNF is surrounded by angle brackets $\langle \rangle$

$\langle \text{symbol} \rangle ::= \text{exp1} \mid \text{exp2} \mid \text{exp3}$
└─ Non Terminal
└─ Sequence of symbols

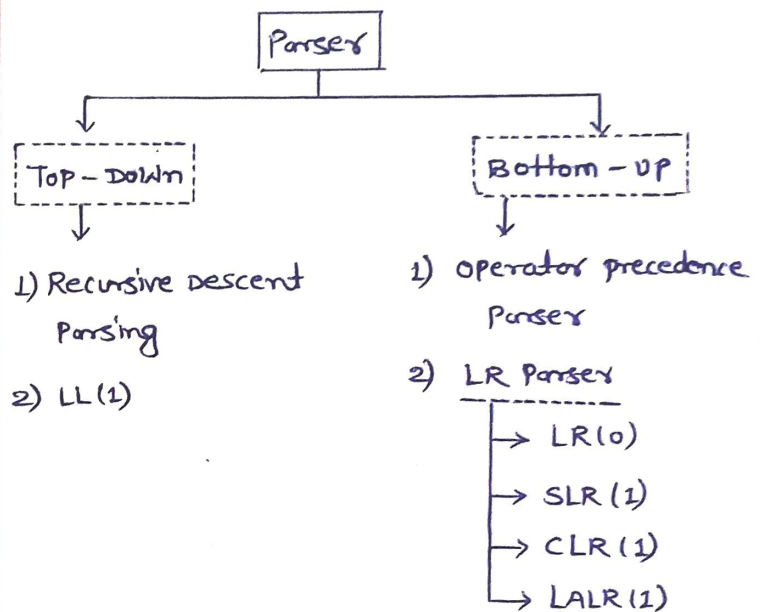
- Widely used in Parsing. helps in building the syntax tree of a program.
- It provides Concise and readable way

⇒ YACC (Yet Another Compiler Compiler)
└─ Unix tool that generates ~~LALR~~ LALR Parsers from CFG (usually in BNF).

* Parsers (Syntax Analyzer)

- main goal is to make sure that the tokens are placed in correct and meaningful order, According to the grammar.

↓
If the structure is correct, Parsers creates a parse Tree / Abstract syntax tree (AST).

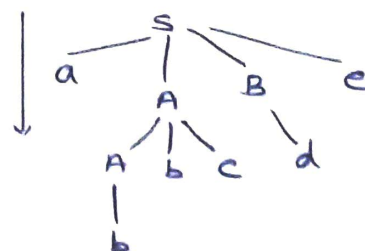


A) Top down Parser

- build a parse Tree from the start symbol (root) down to leaves (End symbol)

$$\begin{cases} S \rightarrow aABE \\ A \rightarrow Abc \mid b \\ B \rightarrow d \end{cases}$$

$W = abbcd$



(B) Bottom-up Parsing -

→ building a parse Tree starting from the leaf Nodes (input symbols) and Working towards the root Node (Start Symbol).

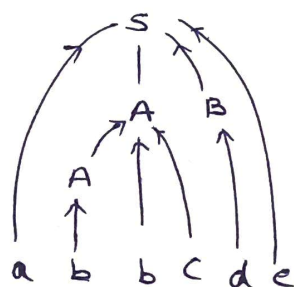
→ Rightmost derivation in Reverse

↳ Parser traces the Right most derivation of the string but Work backwards.

→ often called shift-reduce parser.

$$\begin{cases} S \rightarrow aABE \\ A \rightarrow Abc / b \\ B \rightarrow d \end{cases}$$

$W = abbcd e$



$S \rightarrow aABE$
↓
 $\rightarrow aAde$
↓
 $\rightarrow aAbcde$
↓
 $\rightarrow abbcd e$ (string match)

(1) Recursive Descent Parser -

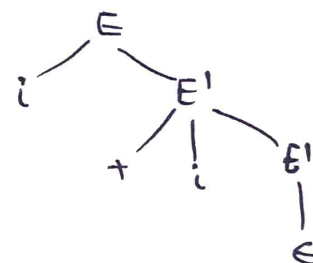
- Top-down Parser
- Processed input based on recursive functions.

Let Grammar -
$$\begin{cases} E \rightarrow iE' \\ E' \rightarrow +iE' / \epsilon \end{cases}$$

```
main()
{
    E();
    if (input == '$')
        success;
    if (input == 'i')
        input++;
}

E'()
{
    if (input == '+') {
        input++;
        if (input == 'i') input++;
        E'();
    }
    else return;
}
```

$W = i + i \$$



• First :-

→ set of terminals that begins in all string derived from A.

1) if $A \rightarrow \alpha a \alpha$, $\alpha \in (V \cup T)^*$
 $\text{first}(A) = a$

2) if $A \rightarrow \epsilon$
 $\text{first}(A) = \epsilon$

3) if $A \rightarrow BC$
 • $\text{first}(A) = \text{first}(B)$
 (if $\text{first}(B)$ doesn't contains ϵ)
 • $\text{first}(A) = \text{first}(B) \cup \text{first}(C)$
 (if $\text{first}(B)$ contains ϵ)

Exa- $S \rightarrow AaB / BA$
 $A \rightarrow a/b$
 $B \rightarrow d/e$

$\text{first}(S) = \{a, b, d, e\}$
 $\text{first}(A) = \{a, b\}$
 $\text{first}(B) = \{d, e\}$

Exa- $S \rightarrow AaB$
 $A \rightarrow b/\epsilon$
 $B \rightarrow c$

$\text{first}(S) = \{b, a\}$
 $\text{first}(A) = \{b, \epsilon\}$
 $\text{first}(B) = \{c\}$

Exa- $S \rightarrow AB$
 $A \rightarrow a/\epsilon$
 $B \rightarrow b/\epsilon$

$\text{first}(S) = \{a, b, \epsilon\}$
 $\text{first}(A) = \{a, \epsilon\}$
 $\text{first}(B) = \{b, \epsilon\}$

• follow :-

→ follow(A) gives set of all terminals that follow immediately to the right of A.

1) if S is on start symbol.
 $\text{follow}(S) = \{\$ \}$

2) if $A \rightarrow \alpha B \beta$
 $\text{follow}(B) = \text{first}(\beta)$
 if $\text{first}(\beta)$ doesn't contains ϵ .

3) $A \rightarrow \alpha B$,
 $\text{follow}(B) = \text{follow}(A)$

Exa- $S \rightarrow aA$
 $A \rightarrow bA/\epsilon$

$\text{follow}(S) = \{\$ \}$
 $\text{follow}(A) = \{\$ \}$

$S \rightarrow AaBb / BbAa$
 $A \rightarrow \epsilon$
 $B \rightarrow \epsilon$

$\text{follow}(S) = \{\$ \}$
 $\text{follow}(A) = \{a\}$
 $\text{follow}(B) = \{b\}$

Exa- $E \rightarrow E+T / T$
 $T \rightarrow T * F / F$
 $F \rightarrow (E) / id$

follow (E) = { \$, +,) }

follow (T) = { \$, +,), * }

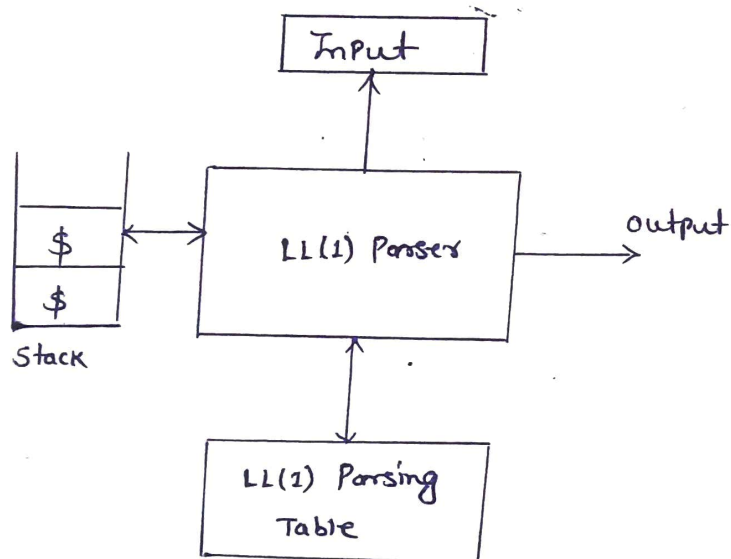
follow (F) = { \$, +,), * }

* LL(1) Parser :-

L : Left to Right Scanning

L : Left most derivation

1 : No. of Look ahead symbols.



LL1 Parser Consist of 3 Components-

- Input Buffer
- Parse Stack
- Parse Table

→ Rule to fill Table

1) Add $A \rightarrow \alpha$ under $m[A, a]$

Where $a \in \text{first}(\alpha)$

2) Add $A \rightarrow \alpha$ under $m[A, a]$

Where $a \in \text{follow}(A)$

If $\text{first}(\alpha)$ Contains ϵ

Example-

	first	follow
$E \rightarrow TE'$	{ id, (}	{ \$,) }
$E' \rightarrow \epsilon / + TE'$	{ ϵ , + }	{ \$,) }
$T \rightarrow FT'$	{ id, (}	{ +, \$,) }
$T' \rightarrow \epsilon / * FT'$	{ ϵ , * }	{ +, \$,) }
$F \rightarrow id / (E)$	{ id, (}	{ *, +, \$,) }

No. of Non Terminals- 5

No. of Terminals = 5

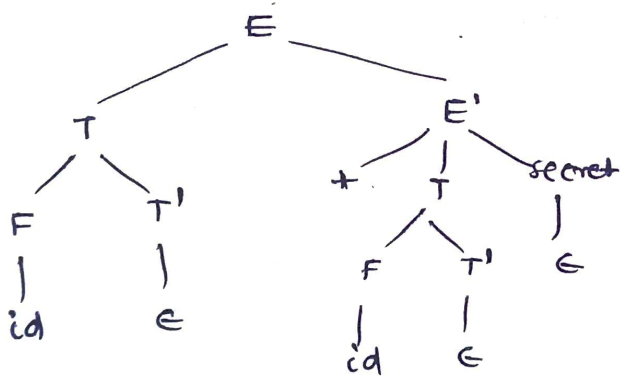
matrix size - $5 * (5+1)$

	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow \epsilon$ $E' \rightarrow + TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

Let input str = "id + id"

Stack	input	Production
\$ E	id + id \$	$E \rightarrow TE'$
\$ E' T	id + id \$	$T \rightarrow FT'$
\$ E' T' F	id + id \$	$F \rightarrow id$
\$ E' T' id	id + id \$	Pop
Same		
\$ E' T'	+ id \$	$T' \rightarrow \epsilon$
\$ E'	+ id \$	$E' \rightarrow +TE'$
\$ E' T +	+ id \$	Pop
\$ E' T	id \$	$T \rightarrow FT'$
\$ E' T' F	id \$	$F \rightarrow id$
\$ E' T' id	id \$	Pop
\$ E' T'	\$	$T' \rightarrow \epsilon$
\$ E'	\$	$E' \rightarrow \epsilon$
\$	\$	Accept

Parse Tree



Algorithm-

Let x is a grammar symbol ~~and~~ (Stack symbol) and a is the look ahead symbol.

if $x == a == \$$, then Parsing is successful.

if $x == a != \$$, then Pop and increment the i/p pointer.

if $x != a != \$$ and $m[x, a]$ contains the production $x \rightarrow abc$, then replace x by abc in reverse order and Continue process.

* Grammar for which LL(1) Parsers Can be Constructed is Known as LL(1) Grammars.

if LL(1) Parser Table Contains multiple entries in the Same cell, then the Grammar is Ambiguous and We Can not solve Ambiguous Grammars with LL(1)

Exa-

$S \rightarrow aBDh$

$B \rightarrow cC$

$C \rightarrow bC/\epsilon$

$D \rightarrow EF$

$E \rightarrow g/\epsilon$

$F \rightarrow f/\epsilon$

$\text{first}(S) = \{a\}$

$\text{first}(B) = \{c\}$

$\text{first}(C) = \{b, \epsilon\}$

$\text{first}(D) = \{g, f, \epsilon\}$

$\text{first}(E) = \{g, \epsilon\}$

$\text{first}(F) = \{f, \epsilon\}$

$\text{follow}(S) = \{\$ \}$

$\text{follow}(B) = \{g, f, h\}$

$\text{follow}(C) = \{g, f, h\}$

$\text{follow}(D) = \{h\}$

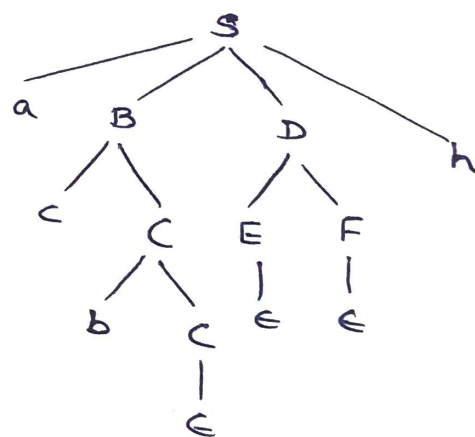
$\text{follow}(E) = \{f, h\}$

$\text{follow}(F) = \{h\}$

	a	b	c	f	g	h	\$
S	$S \rightarrow aBDh$						
B			$B \rightarrow cC$				
C		$C \rightarrow bC$		$C \rightarrow \epsilon$	$C \rightarrow \epsilon$	$C \rightarrow \epsilon$	
D				$D \rightarrow EF$	$D \rightarrow EF$	$D \rightarrow EF$	
E				$E \rightarrow \epsilon$	$E \rightarrow g$	$E \rightarrow \epsilon$	
F				$F \rightarrow f$		$F \rightarrow \epsilon$	

Let input = acbh

Stack	input	Production
\$ S	acbh\$	$S \rightarrow aBDh$
\$ hDBa	acbh\$	Pop
\$ hDB	cbh\$	$B \rightarrow cC$
\$ hDCc	cbh\$	Pop
\$ hDC	bh\$	$C \rightarrow bC$
\$ hDCb	bh\$	Pop
\$ hDC	h\$	$C \rightarrow \epsilon$
\$ hD	h\$	$D \rightarrow EF$
\$ hFE	h\$	$E \rightarrow \epsilon$
\$ hF	h\$	$F \rightarrow \epsilon$
\$ h	h\$	Pop
\$	\$	Accept



* Bottom-up Parser -

- closure(I) :-
- 1) Add LR(0) item I to closure(I)
 - 2) if $A \rightarrow \alpha \cdot B \beta$ is LR(0) item and $B \rightarrow \gamma$ is in G then -
Add $B \rightarrow \cdot \gamma$ to closure(I) Also.
 - 3) Repeat Above 2 steps for Every newly Added LR(0) item.

Exa- $S \rightarrow AA$
 $A \rightarrow aA/b$

$$\text{closure}(S \rightarrow \cdot AA) \Rightarrow \begin{cases} S \rightarrow \cdot AA \\ A \rightarrow \cdot aA \\ A \rightarrow \cdot b \end{cases}$$

- Goto(I, X) :-
- 1) Add LR(0) item I by moving dot after X.
 - 2) Apply closure to the result obtained in step 1.

$$\text{Exa- Goto}(S \rightarrow \cdot AA, A) \Rightarrow \begin{cases} S \rightarrow A \cdot A \\ A \rightarrow \cdot aA \\ A \rightarrow \cdot b \end{cases}$$

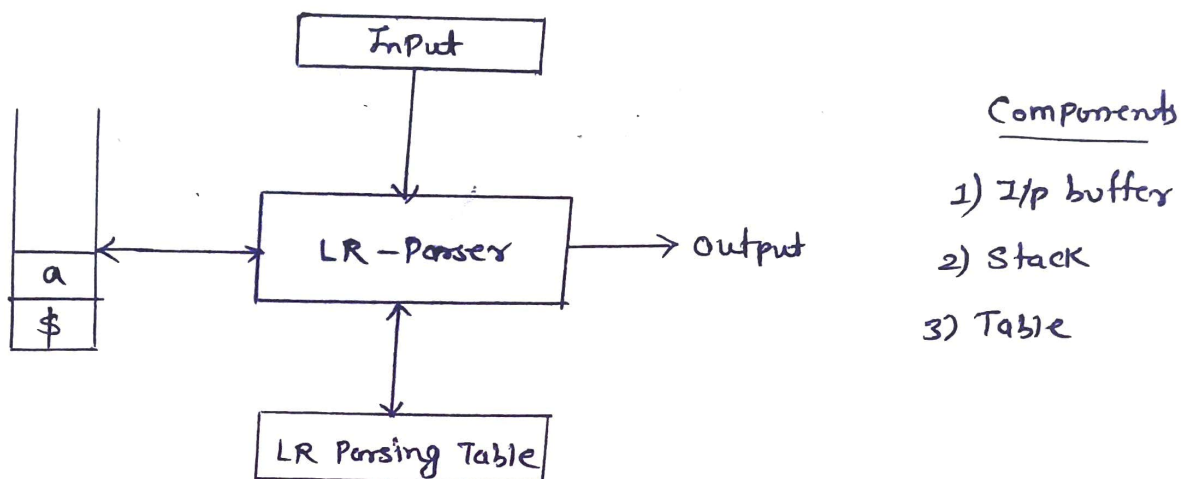
$$\text{Goto}(S \rightarrow A \cdot A, A) \Rightarrow S \rightarrow AA \cdot$$

• Handle - substring of Input string that matches with RHS of any production, is called as Handle.

The process of finding the handle & Replacing that handle by it's LHS variable is called Handle pruning.

→ Bottom-up Parser is also known as shift Reduce Parser.

→ ~~Bottom~~ Bottom up Parser is faster than Top down Parser.



→ operation in shift Reduce Parser-

- 1) Shift :- Shift operation can be used when handle does not occur from the topmost symbol of the stack. Using shift operation, will move a look ahead ~~symbol~~ symbol in stack.
- 2) Reduce - ~~It~~ performs when handle occurs from the topmost symbol from the stack.
- 3) Accept - After scanning the complete I/p string, if the stack contains only the start symbol of the grammar as topmost symbol, then the i/p string is accepted and parsing is successful.
- 4) Error - After the complete I/p string, if the stack contains any symbol which is different from start symbol as a topmost symbol, then parsing is unsuccessful and hence error.

→ Algorithm-

- 1) find Augmented Grammar
- 2) $I_0 = \text{Closure}(\text{Augmented LR}(0) \text{ item})$
- 3) Apply closure and goto function and find all collections of LR(0) item using DFA
- 4) Reduce DFA = LR(0) Parsing table.

Ex 4 -

$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow aA/b \end{aligned} \Rightarrow \begin{aligned} S' &\rightarrow S \\ S &\rightarrow AA \\ A &\rightarrow aA/b \end{aligned}$$

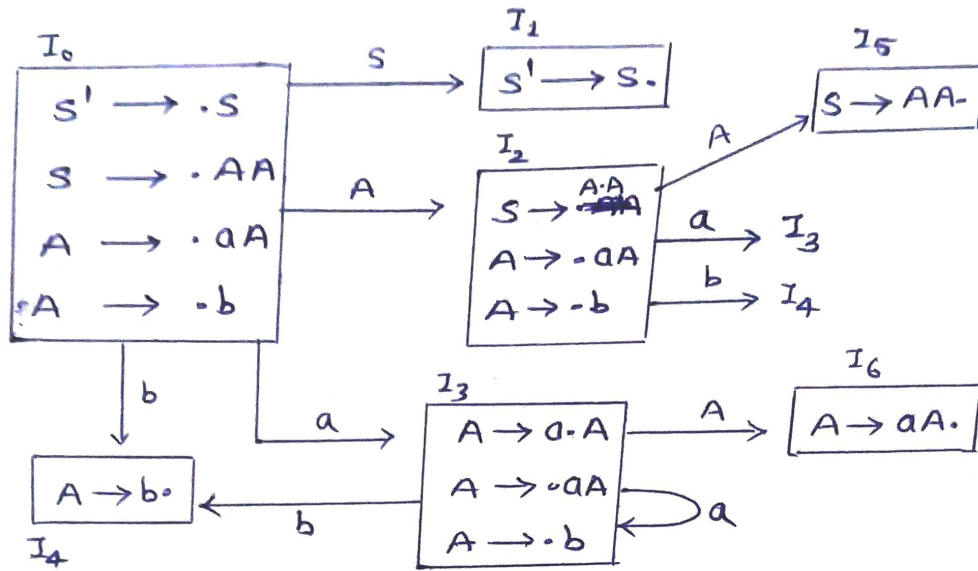


Table-

	Action			Goto	
	a	b	\$	S	A
0	S ₃	S ₄		1	2
1			Accept		
2	S ₃	S ₄			5
3	S ₃	S ₄			6
4	r ₃	r ₃	r ₃		
5	r ₁	r ₂	r ₁		
6	r ₂	r ₂	r ₂		

1) $S \rightarrow AA$

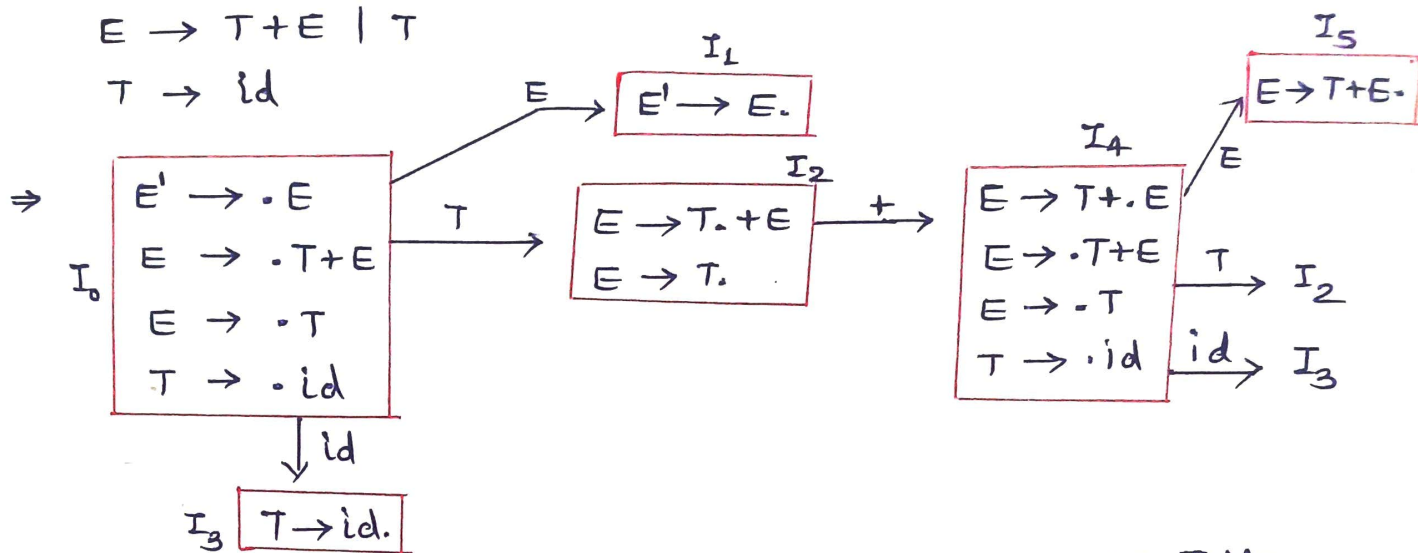
2) $A \rightarrow aA$

3) $A \rightarrow b$

* SLR Parser :- SLR(1)

$$E \rightarrow T + E \mid T$$

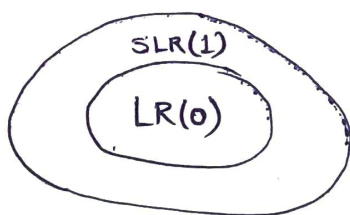
$$T \rightarrow id$$



	Action			Goto	
	id	+	\$	E	T
0	s ₃			1	2
1			Accept		
2	r ₂	s ₄ /r ₂	r ₂		
3	r ₃	r ₂	r ₃		
4	s ₃			5	2
5	r ₁	r ₁	r ₁		

LR(0) Table

Here S-R Conflict Exists, so
Grammar is not LR(0) Grammar



States that have S-R Conflict or
R-R Conflict are called
Inadequate states.

SLR(1) Table

	Action			Goto	
	id	+	\$	E	T
0	s ₃			1	2
1			Accept		
2		s ₄	r ₂		
3		r ₃	r ₃		
4	s ₃			5	2
5			r ₁		

In SLR:-

final item $\rightarrow A \rightarrow B$.
then fill reduce entry
in follow(A)

for I₂ : $E \rightarrow T$.

follow(E) = \$

for I₃ : $T \rightarrow id$.

follow(T) = {+, \$}

Let Expr = id + id \$

0	id	3	T	2	+	4	id	3	T	2
E	5	E	1							

* CLR(1) Parser - (Canonical LR Parser)

→ It uses LookAhead Symbols.

$$LR(1) \text{ items} = LR(0) \text{ items} + \underline{\text{Look Ahead}}$$

if production is $A \rightarrow \alpha \cdot BC, a$
 $\rightarrow B \rightarrow \cdot D$, then LookAhead of $B \rightarrow \cdot D$
 is find by prev. production

$$A \rightarrow \alpha \cdot BC, a$$

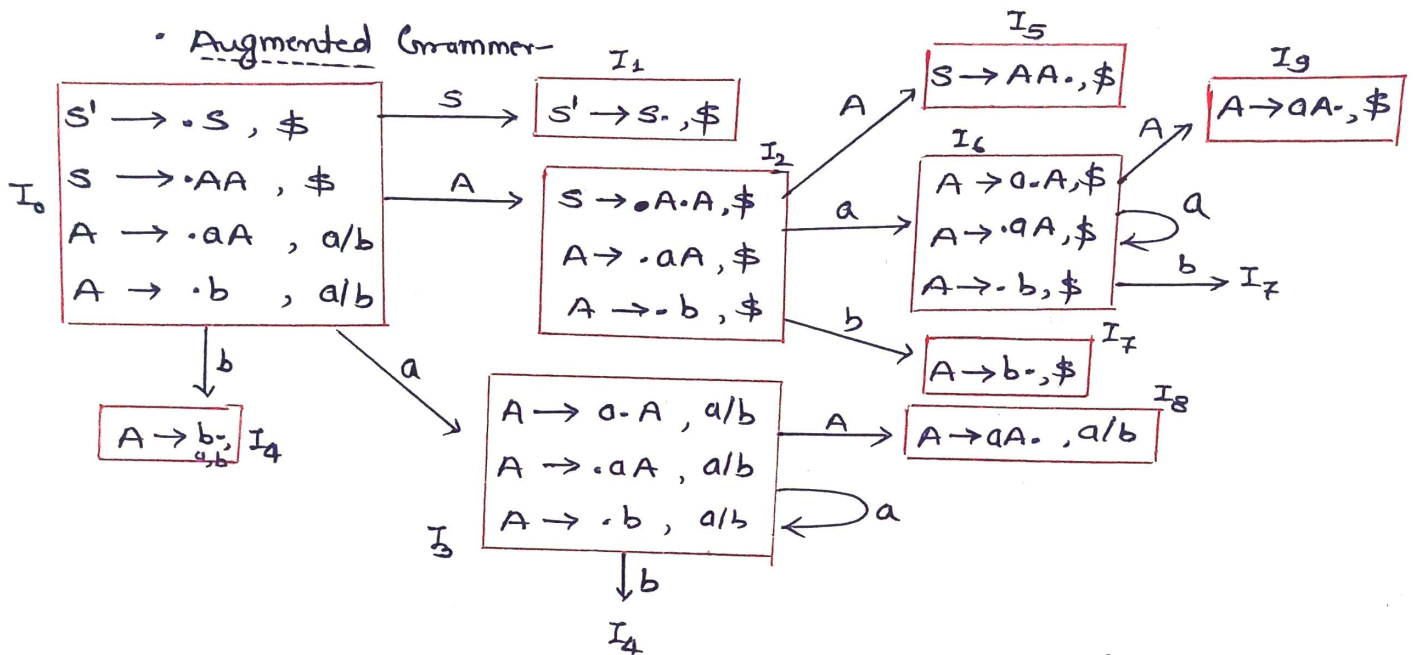
first(C, a)

Exd -

$$S \rightarrow AA$$

$$A \rightarrow aA / b$$

• Augmented Grammar

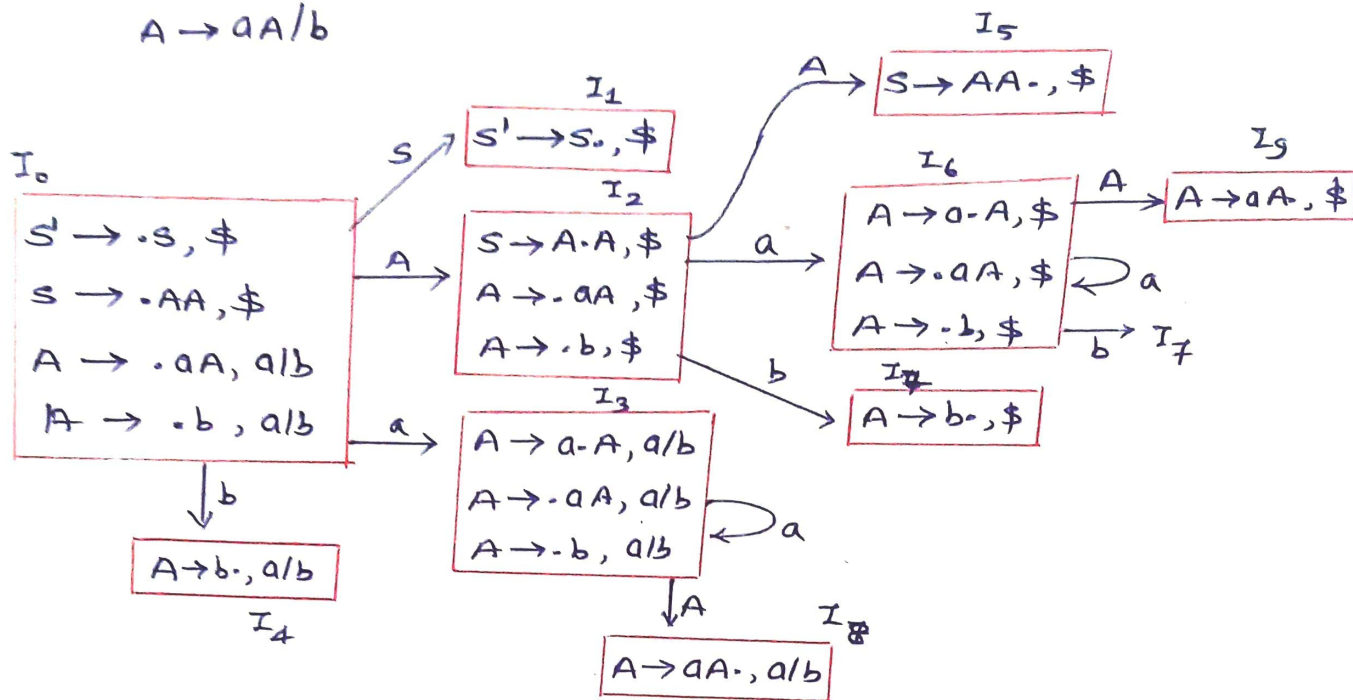


	Action			Goto	
	a	b	\$	S	A
0	S ₃	S ₄		1	2
1			Accept		
2	S ₆	S ₇			5
3	S ₃	S ₄			8
4	r ₃	r ₃			
5			r ₁		
6	S ₆	S ₇			9
7			r ₃		
8	r ₄	r ₄			
9			r ₄		

* LALR(1) :-

- Similar to the CLR, It reduced the size of parsing Table of CLR, Combining the similar states of CLR parsing Table.

Exa- $S \rightarrow AA$
 $A \rightarrow aA/b$



$\Rightarrow$ I_3 and I_6 are similar, (I_4 , and I_7) are also similar
 I_8 and I_9)

	Action			Goto	
	a	b	\$	S	A
0	S_3	S_4		1	2
1			Accept		
2	S_6	S_7			5
3	S_3	S_4			8
4	r_3	r_3			
5			r_1		
6	S_6	S_7			9
7			r_3		
8	r_2	r_2			
9			r_2		

CLR Table

	a	b	\$	S	A
36	S_{36}	S_{47}			89
47	r_3	r_3	r_3		
89	r_2	r_2	r_2		

(Replace 3, 6, 4, 7, 8, 9 by these 3 Rows)

* operator precedence Parser-

• Operator Grammar-

A Grammar G does not contain:

1.) NULL Production

2.) 2 Adjacent variable on RHS of production

$$A \rightarrow \epsilon \quad X$$

$$A \rightarrow BCD \quad X$$

$$A \rightarrow BC \quad X$$

$$E \rightarrow E + E \mid E * E \mid id \quad \checkmark$$

Exa- $P \rightarrow SR \mid S$

$$R \rightarrow bSR \mid bS$$

$$S \rightarrow WbS \mid W$$

$$W \rightarrow L * W \mid L$$

$$L \rightarrow id$$

$$\Rightarrow P \rightarrow S bSR \mid S bS \mid S$$

$$R \rightarrow bP \mid bS \quad \leftarrow \text{(useless)}$$

$$S \rightarrow WbS \mid W$$

(Operator Grammar)

$$W \rightarrow L * W \mid L$$

$$L \rightarrow id$$

Let Grammar- $E \rightarrow E + E \mid E * E \mid id$

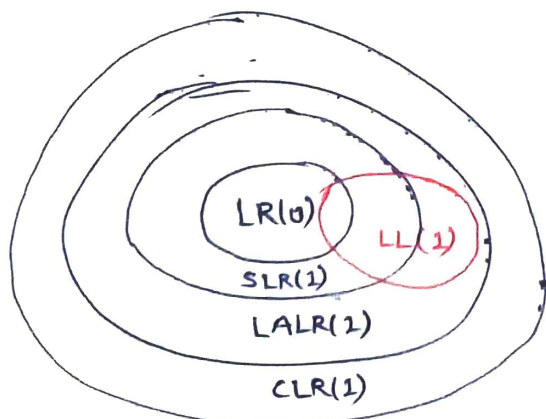
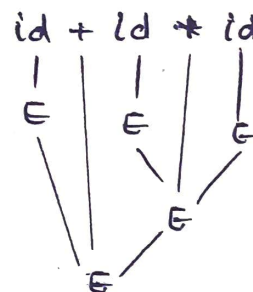
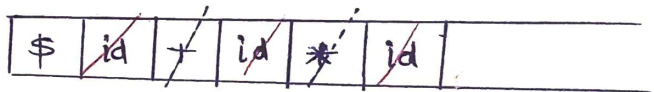
m	id	*	+	\$
id	-	>	>	>
*	<	>	>	>
+	<	<	>	>
\$	<	<	<	-

Operator Relation Table

$id > (*, +, \$)$

$* > *$ (Associativity)

str = id + id * id \$



LR(0) SLR(1) LALR(1) CLR(1)

$$n_1 = n_2 = n_3 \leq n_4$$