* Push down Automata (PDA)

- FA + stack
- tuple = $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Q : Not Empty finite set of states

Σ : Input variables

Γ : Stack Alphabet

δ : Transition function

q_0 : Initial state

Z_0 : Stack start symbol

F : set of final states

for Deterministic PDA (DPDA) :-

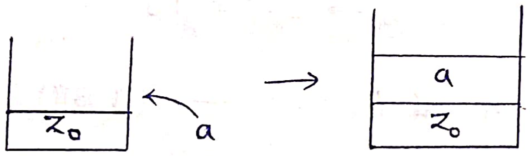
$$\delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \longrightarrow Q \times \Gamma^*$$

for Non-Deterministic PDA (NPDA):

$$\delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \longrightarrow 2^{Q \times \Gamma^*}$$

- stack operation

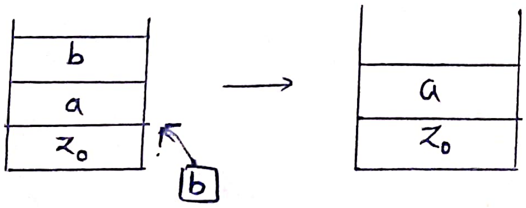
1) Push operation



$$(q_0, a, Z_0) \longrightarrow (q_0, aZ_0)$$

↑ I/P symbol ↑ Top of stack

2) Pop operation



$$(q_0, b, b) \longrightarrow (q_1, \epsilon)$$

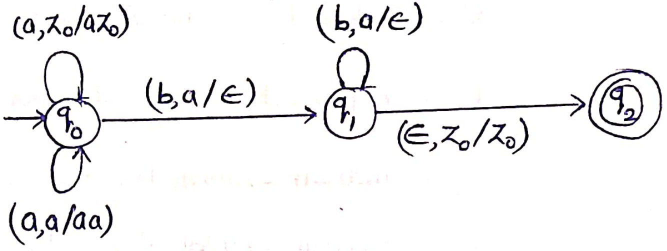
3) Skip operation

$$(q_0, a, b) \longrightarrow (q_0, b)$$

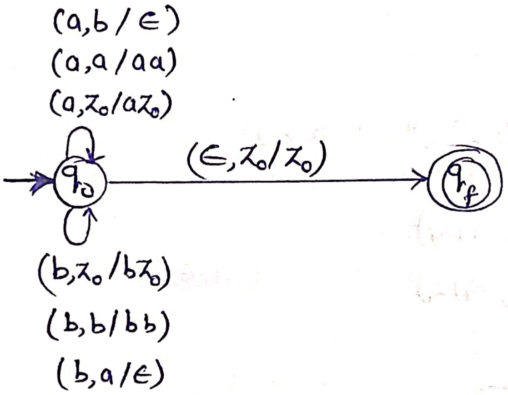
Exa

1) $L = a^n b^n \mid n \geq 1$

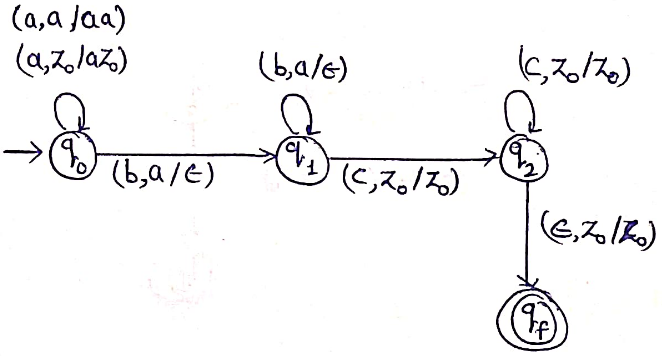
$L = \{ab, aabb, aaabbb \text{ ------ }\}$



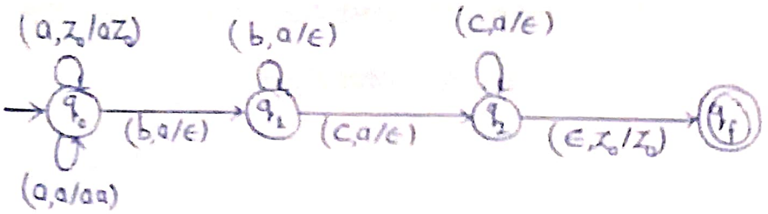
2) $L = \{w \mid n_a(w) = n_b(w)\}$



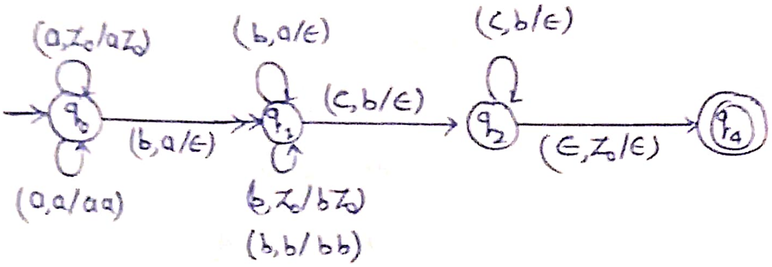
3) $L = a^n b^n c^m \mid n, m \geq 1$



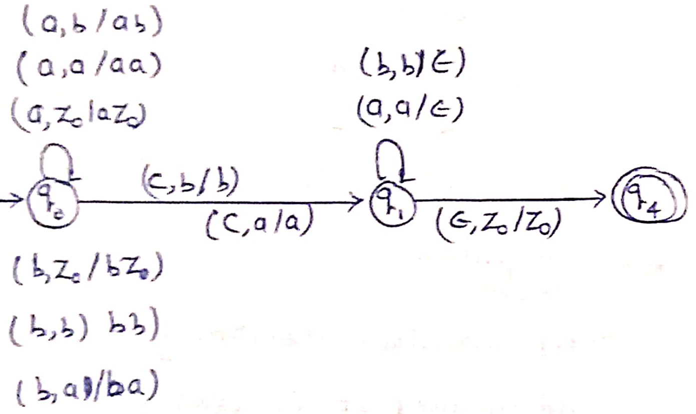
4) $L = a^{m+n} b^{m} c^{n} \mid m,n \geq 1$



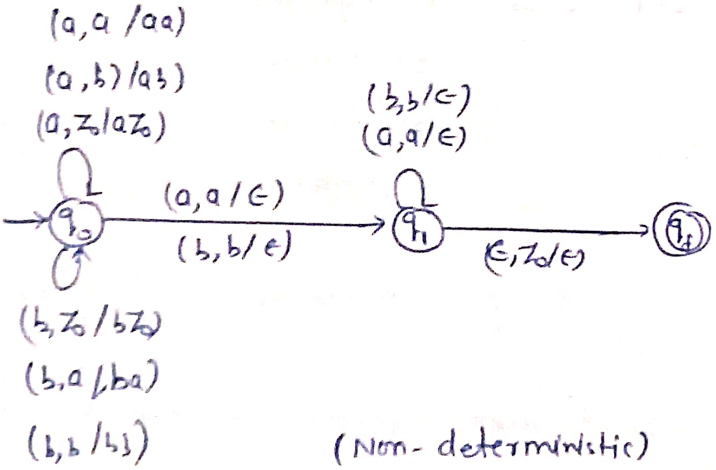
5) $L = a^{n} b^{m+n} c^{m} \mid m,n \geq 1$



6) $L = \{ wcw^{R} \mid w \in \{a,b\}^{+} \}$



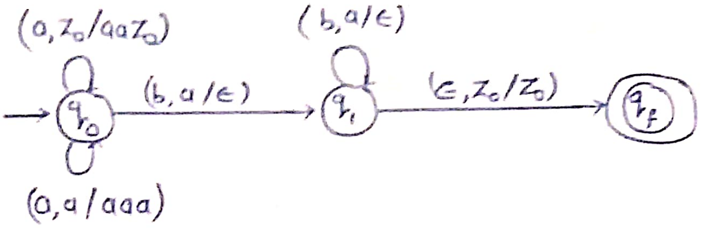
7) $L = ww^{R}, \quad w \in \{a,b\}^{+}$



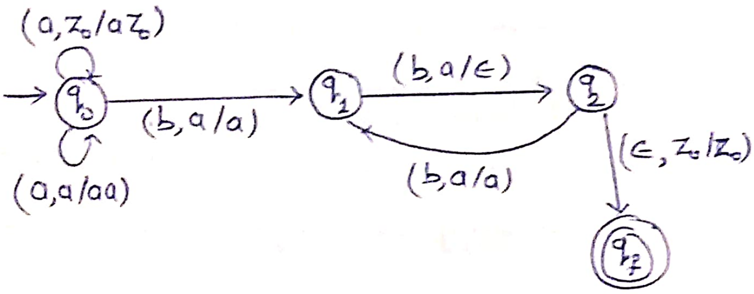
(Non-deterministic)

8) $L = a^{n} b^{2n} \mid n \geq 1$

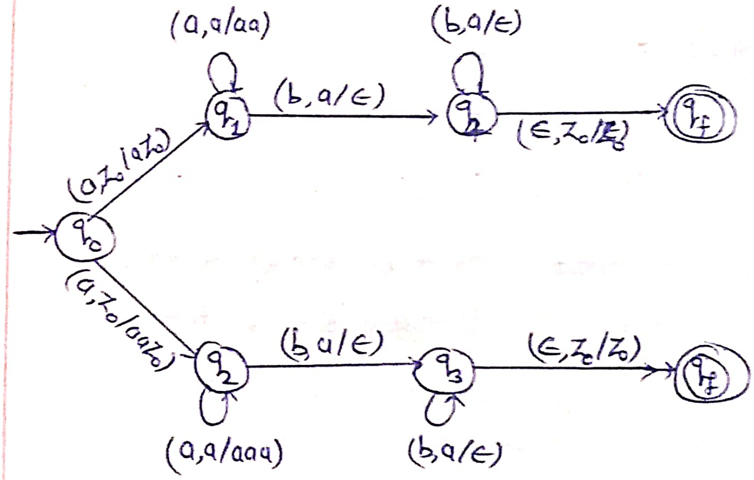
(A) Pushing "aa"



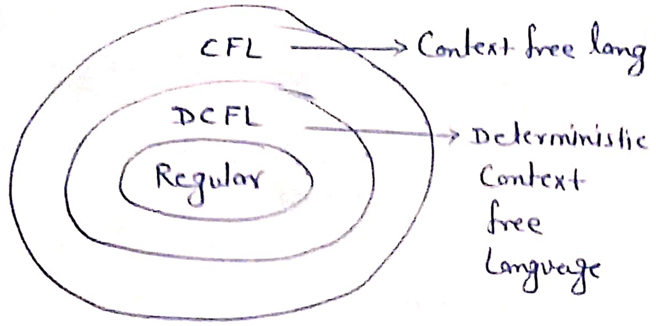
(B) skip 'b' first, then pop 'a' by next 'b'



9) $L = \{ a^{n} b^{n} \mid n \geq 1 \} \cup \{ a^{n} b^{2n} \mid n \geq 1 \}$



* check Wheather a language is CFL or not?



1.) $a^n b^n \mid n \geq 1$
↳ There is a Condition on a, b ⇒ RL ✗
↳ We Can design DPDA ⇒ DCFL ✓
↳ So It is CFL ✓

2) $a^{m+n} b^n c^m \mid m,n \geq 1$
RL ✗ , DCFL ✓ , CFL ✓

3) $a^m b^m c^n d^n \mid m,n \geq 1$
RL ✗ , DCFL ✓ , CFL ✓

4) $a^m b^n c^m d^n \mid m,n \geq 1$
RL ✗ , CFL ✗

5) $a^m b^i c^m d^j \mid m,i,j \geq 1$
RL ✗, DCFL ✓, CFL ✓

6) $a^i b^j c^K \mid K = i+j$
RL ✗ , DCFL ✓ , CFL ✓

7) $a^i b^j c^K d^l \mid i = K$ and $j = l$
RL ✓, CFL ✗

8) $a^i b^j c^K d^l \mid i,j,k,l \geq 1$
RL ✓ , CFL ✓,

9) $a^n b^{2n} \mid n \geq 1$
RL ✗, DCFL ✓, CF✓

10) $a^n b^n c^n \mid n \geq 1$
CFL ✗

11.) $a^n b^n c^n d^n \mid n \geq 1$
RL ✗, CLF ✗

12) $wcw^R \mid w \in \{a,b\}^+$
RL ✗, DCFL ✓, CFL ✓

13) $ww^R \mid w \in \{a,b\}^+$
RL ✗ DCFL ✗ CFL ✓

14) $ww \mid w \in \{a,b\}^+$
CFL ✗

15) $a^{n^2} \mid n \geq 1$
CFL ✗

16) $a^{2^n} \mid n \geq 1$ ⇒ CLF ✗
(Not AP series)

17) $a^n b^{n^2} \mid n \geq 1$ ⇒ CFL ✗

18) $a^n b^{2^n} \mid n \geq 1$ ⇒ CFL ✗

19) $a^n b^{2n} c^{3n} \mid n \geq 1$ ⇒ CFL ✗

20) $n_a(w) = n_b(w) = n_c(w)$
↳ CFL ✗
($a^n b^n c^n$ also not CFL)

* PDA to CFG Conversion :-

∵ PDA $= (Q, \Sigma, \delta, \Gamma, q_0, Z_0, F)$

$\downarrow$

CFG $= (V, T, P, S)$

Rules -

$S \cup [q, A, p]$, $q, p \in Q$, $A \in \Gamma$

1) $S \longrightarrow [q_0, Z_0, p]$ for each p

2) if $\delta(q, x, A) = (p, B_1 B_2 \ldots\ldots B_m)$

$[q, A, q_{m+1}] \longrightarrow x [p, B_1, q_2] [q_2, B_2, q_3] \ldots$

$\ldots [q_m, B_m, q_{m+1}]$

$x \in \Sigma \cup \{\epsilon\}$

3) if $\delta(q, x, A) = (p, \epsilon)$

$[q, A, p] \rightarrow x$

Exa-

1.) $\delta(q_0, a, Z_0) \longrightarrow (q_0, XZ_0)$

2) $\delta(q_0, a, X) = (q_0, XX)$

3) $\delta(q_0, b, X) = (q_1, \epsilon)$

4) $\delta(q_1, b, X) = (q_1, \epsilon)$

5) $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$m = (\{q_0, q_1\}, \{a, b\}, \delta, \{Z_0, X\}, q_0, Z_0, \phi)$
$\quad Q \qquad \Sigma \qquad \Gamma \qquad F$

CFG-

$S \rightarrow [q_0, Z_0, q_0]$, $S \rightarrow [q_0, Z_0, q_1]$

① $[q_0, Z_0, q_0] \longrightarrow a [q_0, X, q_0] [q_0, Z_0, q_0]$ ✗

$[q_0, Z_0, q_0) \longrightarrow a (q_0, X, q_1] (q_1, Z_0, q_0]$ ✗

$[q_0, Z_0, q_1] \longrightarrow a [q_0, X, q_0] [q_0, Z_0, q_1]$ ✗

$[q_0, Z_0, q_1] \rightarrow a [q_0, X, q_1] [q_1, Z_0, q_1]$

② $[q_0, X, q_0] \rightarrow a [q_0, X, q_0] [q_0, X, q_0]$ ✗

$[q_0, X, q_0] \rightarrow a [q_0, X, q_1] [q_1, X, q_0]$ ✗

$[q_0, X, q_1] \rightarrow a [q_0, X, q_0] (q_0, X, q_1]$ ✗

$[q_0, X, q_1] \rightarrow a [q_0, X, q_1] [q_1, X, q_1]$

3. $(q_0, X, q_1) \longrightarrow b$

4. $[q_1, X, q_1] \longrightarrow b$

5. $[q_1, Z_0, q_1] \longrightarrow \epsilon$

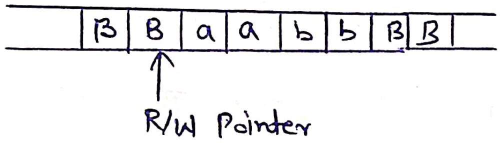
Useless Production-

$(q_1, Z, q_0]$

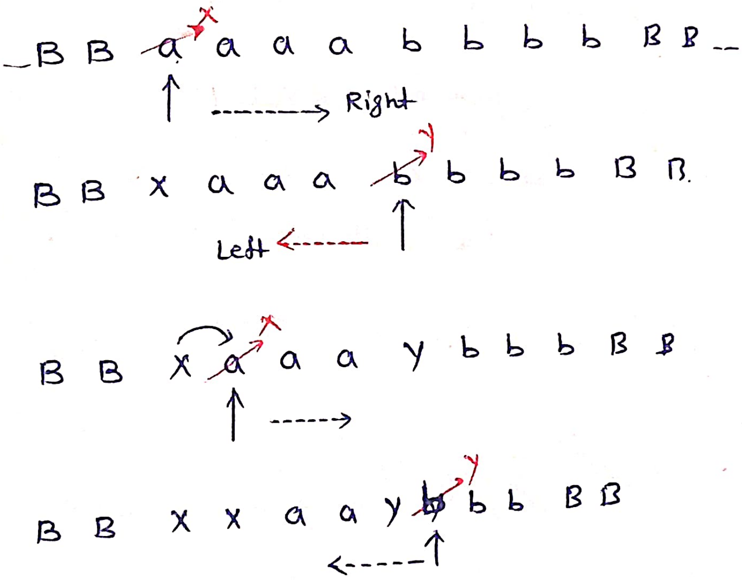
$[q_1, X, q_0]$

* Turing Machine

- → A Turing m/c Consists of a tape of infinite length on Which read and Writes operation Can be performed.
- → The Tape Consists of infinite cells on Which each cell either Contains input symbol or a special symbol Called Blank.
- → It also Consists of a head pointer Which Points to cell currently being read and it Can move in both directions.



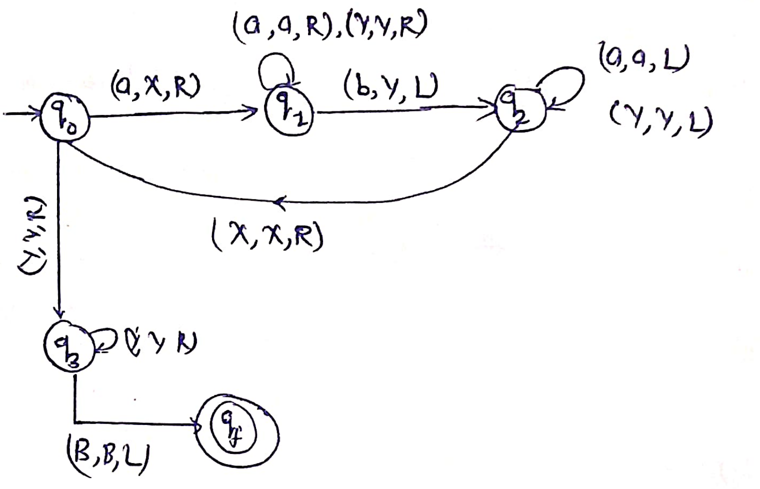
$L = a^n b^n \mid n \geq 1$



Repeat process

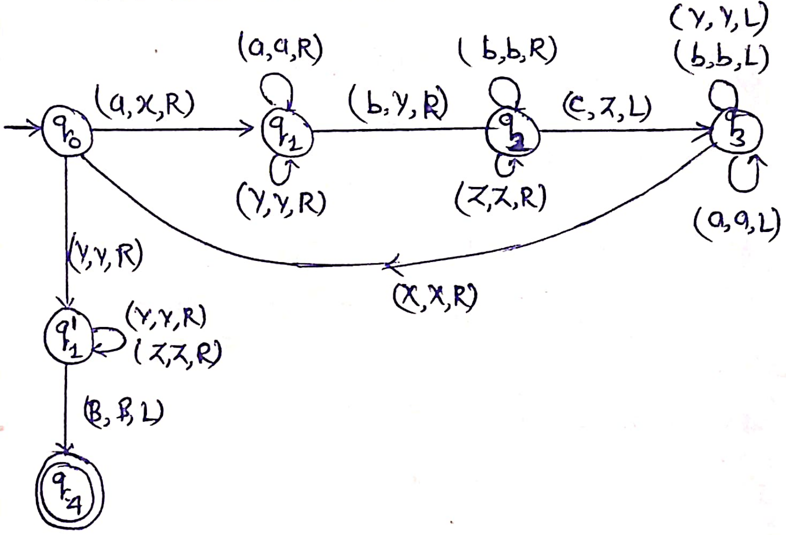
B B X X X X Y Y Y Y B B

δ = (input, output, Head movement)

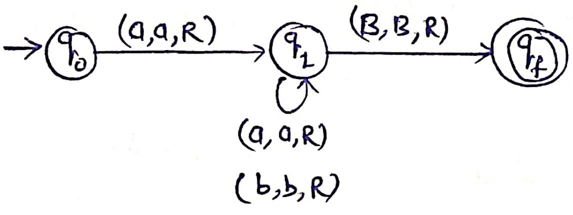


2) $L = a^n b^n c^n \mid n \geq 1$

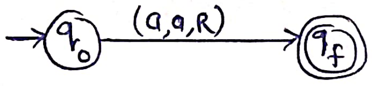
----- B B a a a b b b c c c B B ---



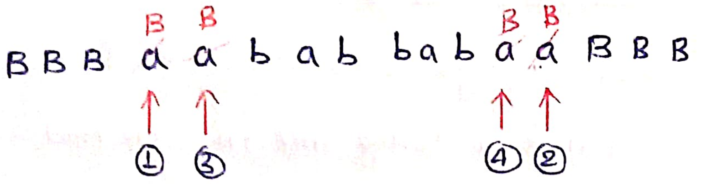
3) L = ~~a^n +/+~~ $a(a+b)^*$

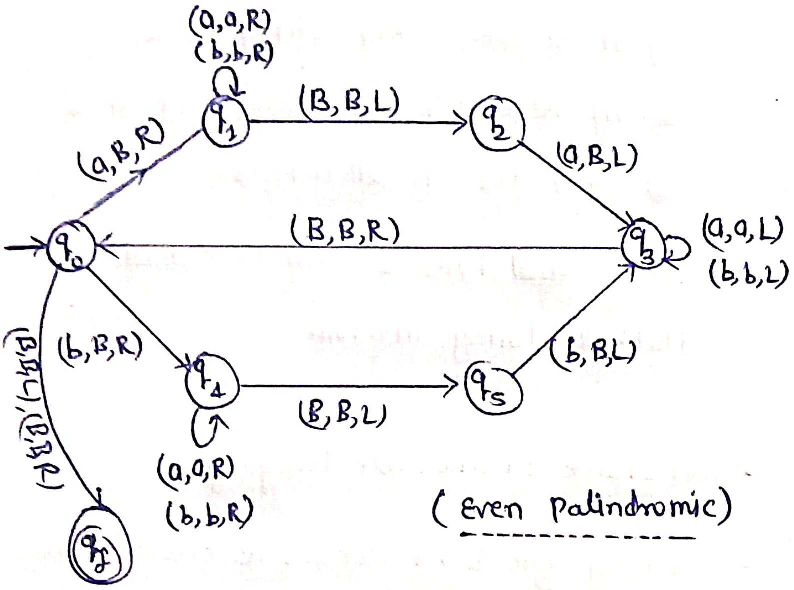


or



4) $L = ww^R$. $w \in \{a,b\}^*$



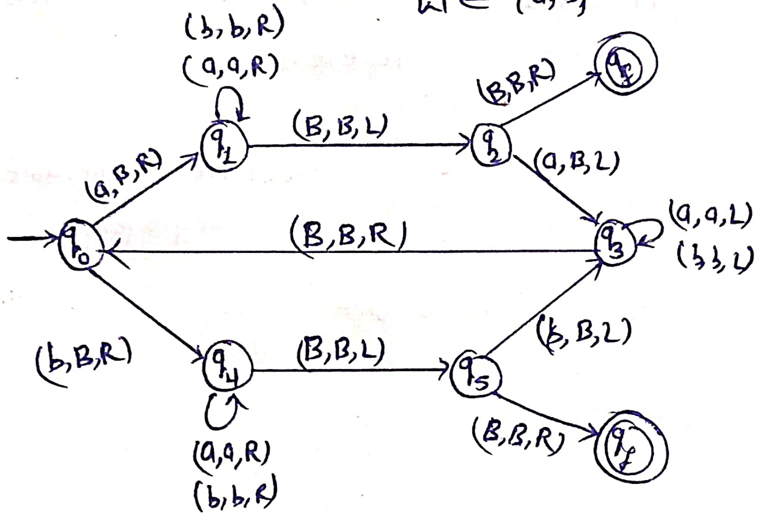


(even palindromic)

5) odd palindrome -

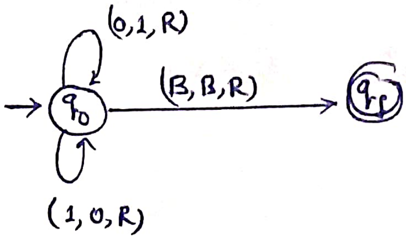
$L = waw^R$ or wbw^R

$w \in \{a,b\}^*$

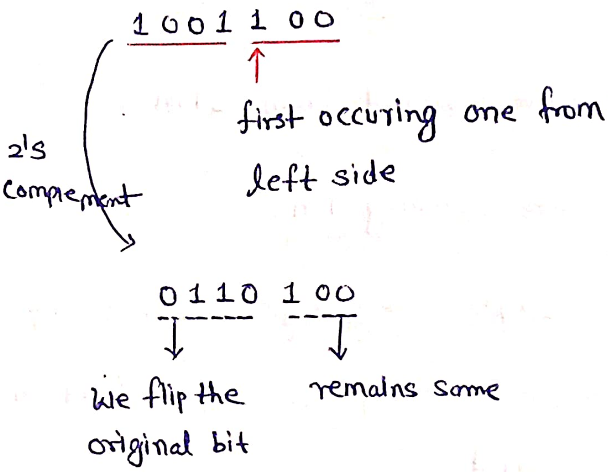


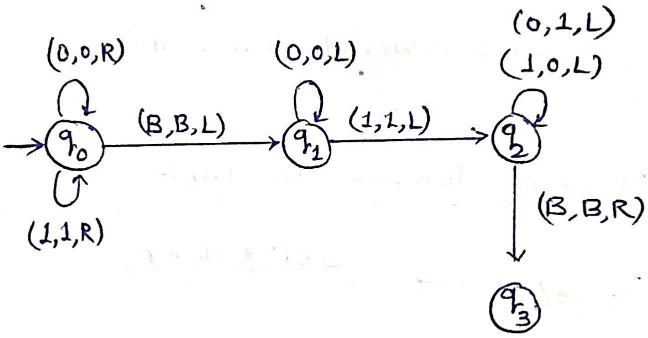
6) $L = wcw$ | $w \in \{a,b\}^*$

7) Tm for 1's Complement



8) Tm for 2's Complement





9) Tm As an Adder

a = 4 , b = 5

Convert Numbers into unary (we use only one symbol)

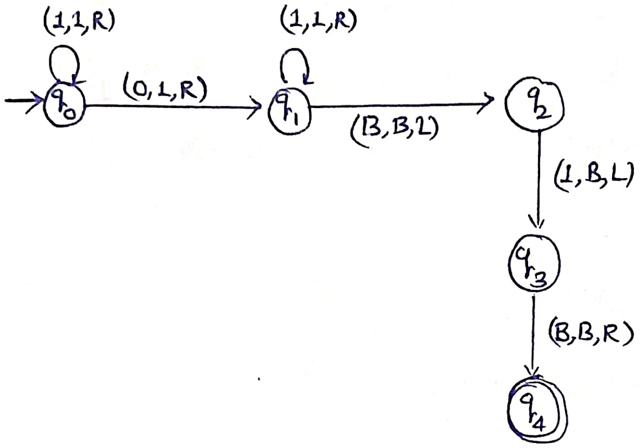
a = 4 = 1 1 1 1

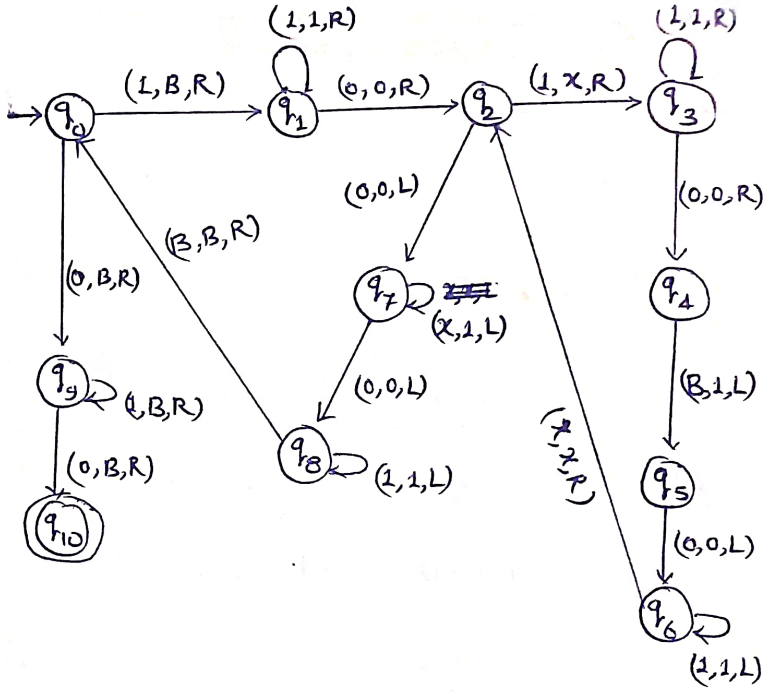
b = 5 = 1 1 1 1 1

Write this on a tape

$$B\ B\ \underbrace{1\ 1\ 1\ 1}_{a}\ 0\ \underbrace{1\ 1\ 1\ 1\ 1}_{b}\ B\ B$$

$B\ B\ 1\ 1\ 1\ 1\ \overset{1}{\cancel{0}}\ 1\ 1\ 1\ 1\ \overset{B}{\cancel{1}}\ B\ B$





10) TM ~~As~~ ~~e~~ for multiplication–

$x = 2 \qquad y = 3$

$x = 11 \qquad y = 111$

$\Rightarrow B\ B\ \overset{B}{\cancel{1}}1\ 0\ 111\ 0\ BBB\ BBB\ ---$

$B\ B\ B\ 1\ 0\ \overset{x}{\cancel{1}}11\ 0\ \overset{1}{\cancel{B}}BB\ BBB$

$B\ B\ B\ 1\ 0\ x\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\ 0\ 11\ \overset{1}{\cancel{B}}\overset{1}{\cancel{B}}B\ BB$

$B\ B\ B\ 1\ 0\ \overset{1}{\cancel{x}}\overset{1}{\cancel{x}}\overset{1}{\cancel{x}}\ 0\ 111\ BBB$

← Left move

Again Repeat process and at the End 6's time 1 Print at the Right hand side in place of Blank

(11) TM As an Comparator.

$a = 3\ ,\quad b = 3$

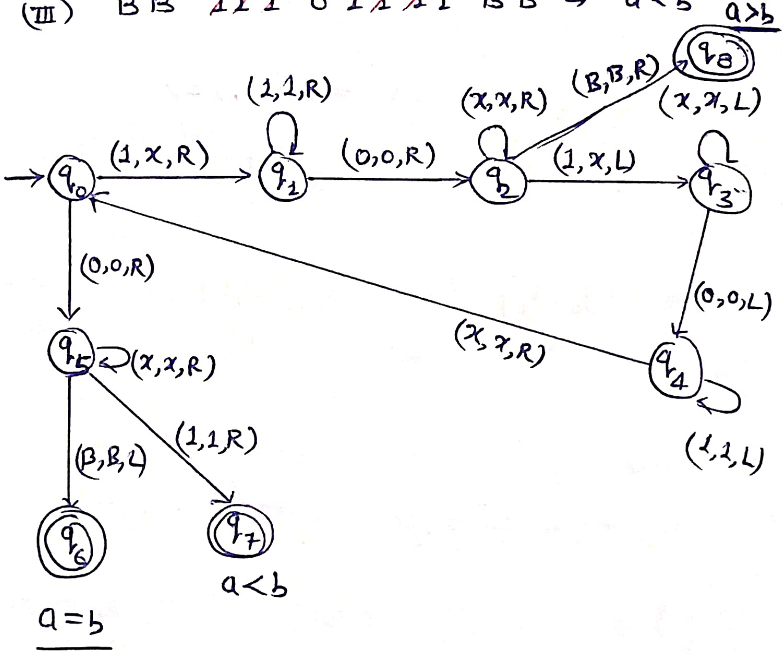
$B\ B\ \overset{x}{\cancel{1}}\ 1\ 1\ \ 0\ \ \overset{x}{\cancel{1}}\ 1\ 1\ \ B\ B$

$B\ B\ x\ \overset{x}{\cancel{1}}\ 1\ \ 0\ \ x\ \overset{x}{\cancel{1}}\ 1\ \ B\ B$

(I) $B\ B\ \overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\ 0\ \overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\ BB \Rightarrow a = b$

(II) $B\ B\ \overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}1\ 0\ \overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\ BB \Rightarrow a > b$

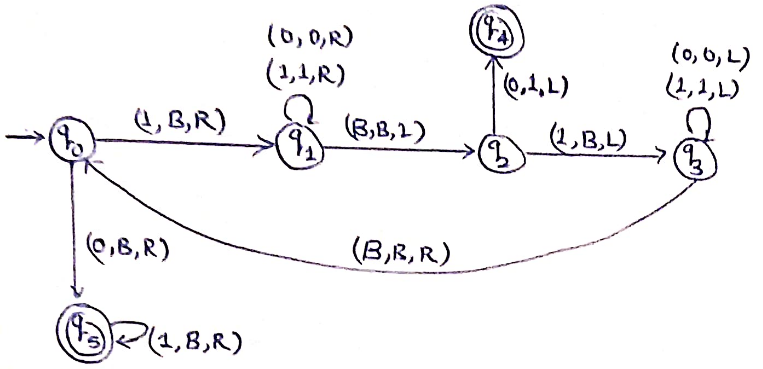
(III) $B\ B\ \overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\ 0\ \overset{x}{\cancel{1}}\overset{x}{\cancel{1}}\overset{x}{\cancel{1}}1\ BB \Rightarrow a < b$



(12) Tm ~~As~~ for a Subtraction :

$$f(m,n) = \begin{cases} m-n, & m>n \\ 0, & m\leq n \end{cases}$$

BB 1111 0 11 BB



~~m=n~~ O/P=0

- **Formal definition –**

$M(Q, \Sigma, \delta, \Gamma, q_0, B, F)$

Q : Non Empty finite set of states

Σ : Input Alphabet

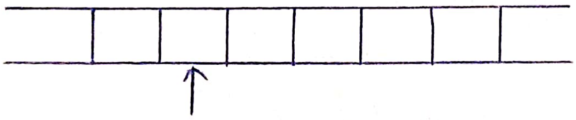
δ : Transition function

Γ : Tape Alphabet

q_0 : Initial state $q_0 \in Q$

B : special symbol Called Blank

F : set of final states. $F \subseteq Q$



Read /Write Head

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L,R\}$$

$$\Gamma = \{a,b,x,y,B\}$$

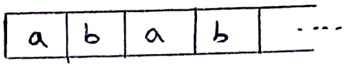
$$\Sigma \subseteq \Gamma - \{B\}$$

✱ Variations of Turing machine :-

1) Tm With Stay option :

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L,R,S\}$$

2) Tm With Semi Infinite Tape :

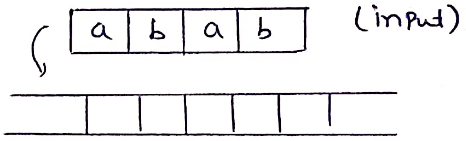


↳ This also have Same power

3) OFFline Tm :-

→ In This We have Read only input tape for separate input storage.

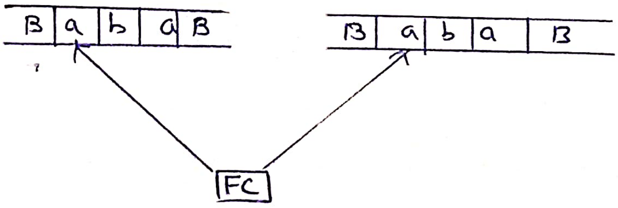
→ The machine Can not modify the input.



4) multi Tape Tm :-

→ It has multiple tapes With own read Write head.

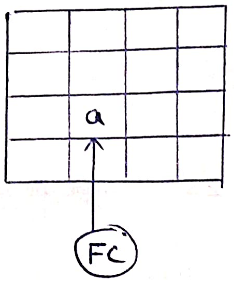
→ A single finite Control that reads all heads Simultaneously.



→ ~~[illegible]~~ Not more PoWerful but Efficient than Standard Tm.

4) multi-dimensional Turing m/c -

$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R, U, D\}$



→ Same Powerful than STm

5) ~~Jum~~ Jumping Tm :-

$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \{L, R\} \times \{n\}$

→ It Can Jumps n cells to left or Right

→ same Powerful than STm

6) Non - deterministic Tm :-

$\delta : Q \times \Gamma \longrightarrow 2^{Q \times \Gamma \times \{L, R\}}$

→ Deterministic and Non Deterministic Tm Contain Same Power.

DTm $\cong$ NDTm

- Recursive Language -

→ A Language L is recursive if there exists a Turing m/c m such that -

1) m Accepts Every String in L
2) m rejects Every string not in L
3) m halts on all inputs.

→ such ~~Kind~~ type of Tm is Called Halting Turing machine.

- Recursive Enumerable Languag :-

→ A Language L is Recursive Enumerable Language If there Exists a Turing machine m such that :-

if $w \in L \Rightarrow$ m halts and Accepts

if $w \notin L \Rightarrow$ m may Reject or loop forever.