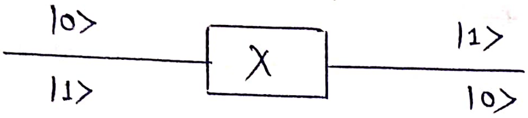
* Quantum Gates :-

1) Pauli-x Gate :-

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Matrix Representation of a gate –

$$A = \sum_i |\text{input}_i\rangle \langle \text{output}_i|$$



$$\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$$

↳ Quantum Not gate

2) Pauli-y Gate :-

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

3) Pauli-z gate :-

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$z|0\rangle = |0\rangle$$

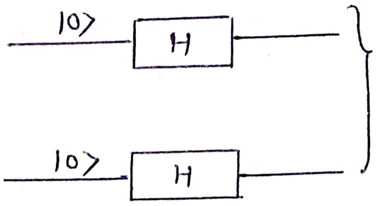
$$z|1\rangle = -|1\rangle \quad \text{(Phase change)}$$

4) Hadmard Gate :-

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1| \right].$$

$$H|0\rangle = |+\rangle$$



$$\psi_f = H|0\rangle \otimes H|0\rangle$$

$$\psi_f = \frac{1}{(\sqrt{2})^2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

5) Rotational Gate :-

$$\because e^{i\eta U} = \cos\eta\, I - i\sin\eta\, U$$

$$R_x(\eta) = e^{-i\eta X/2} = \cos\left(\frac{\eta}{2}\right)I - i\sin\left(\frac{\eta}{2}\right)X$$

$$= \begin{bmatrix} \cos(\eta/2) & -i\sin\left(\frac{\eta}{2}\right) \\ -i\sin\left(\frac{\eta}{2}\right) & \cos\left(\frac{\eta}{2}\right) \end{bmatrix}$$

$$R_y(\eta) = e^{-i\eta \frac{Y}{2}} = \cos\left(\frac{\eta}{2}\right)I - i\sin\left(\frac{\eta}{2}\right)Y$$

$$= \begin{bmatrix} \cos(\eta/2) & -\sin(\eta/2) \\ \sin(\eta/2) & \cos(\eta/2) \end{bmatrix}$$

$$R_z(\eta) = e^{-i\eta Z/2} = \cos(\eta/2)I - i\sin(\eta/2)Z$$

$$= \begin{bmatrix} e^{-i\eta/2} & 0 \\ 0 & e^{i\eta/2} \end{bmatrix}$$

Adding a global phase $e^{i\eta/2}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\eta} \end{bmatrix}$$

Case-1 : $\eta = 2\pi$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Case-2 : $\eta = \pi/2$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}$$

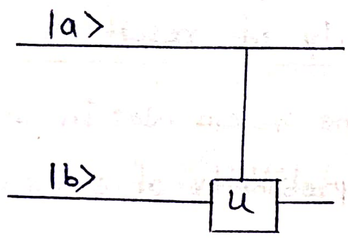
$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Case-3 : $\eta = \pi/4$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

$$T = \sqrt{S}$$

* 2 qubit quantum Gate :-



Controlled unitary operator

$$CU = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & u_2 \end{pmatrix}$$

$u_2 = X \Rightarrow$ C-Not

$u_2 = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow C\text{-}Z$

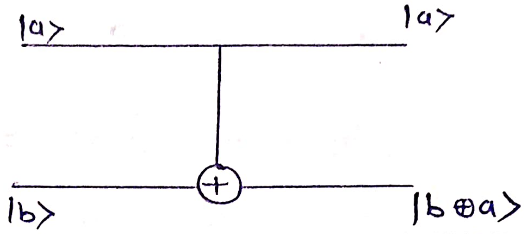
$u_2 = S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \Rightarrow$ C- Phase

C-Not

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$|10\rangle \longrightarrow |11\rangle$

$|11\rangle \longrightarrow |10\rangle$



modulo 2 Add

$0+0 = 0$

$1+1 = 0$

$0+1 = 1$

$1+0 = 1$

Swap Gate -

$|00\rangle \longrightarrow |00\rangle$

$|10\rangle \longrightarrow |01\rangle$

$|01\rangle \longrightarrow |10\rangle$

$|11\rangle \longrightarrow |11\rangle$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$