* Quantum Computing *

- **Bit :-**
  - → A digital Computers stores and processes information using bits, Which Can be either 0 or 1.

- **Qubit :-**
  - → Physical Carrier of quantum information.
  - → qubit Can be in any one of two states as well as in super posed state simultaneously.

    0, 1 or both in Same time

* The difference between qubits and Classical bits is that a qubit Can be in linear Combination (super position) of two states $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

→ $\alpha, \beta \in$ Complex Number

→ $|\alpha|^2$ is the probability to find $|\psi\rangle$ in state $|0\rangle$ (i.e. ground state) or $|\beta|^2$ is the probability to find $|\psi\rangle$ in state $|1\rangle$ (i.e. Excited state)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

→ **Properties of qubits :-**

1) Qubits make use of discrete energy state Particles such as electrons and photons.

2) Qubits exists in two quantum states $|0\rangle$ and $|1\rangle$ or in a linear Combination of both states. This is Known as super position.

3) Unlike Classical bits, qubit Can work with the overlap of both 0 & 1 states. for example a 4 bit register Can store one Number from 0 to 15, but 4-qubit register Can store all 16 Numbers

4) When the qubit is measured, It Collapses to one of the two basis states $|0\rangle$ or $|1\rangle$

5) Quantum entanglement and quantum tunneling are two exclusive Properties of qubit

6) State of qubit is represented using Bloch sphere

* Dirac Representation:-

$| \rangle \rightarrow$ Ket Notation

$\langle |$ $\rightarrow$ brac Notation

→ Ket vector is Column vector

→ brac vector is Row vector.

↳ Transpose of Congvege Elements of Ket

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha|0\rangle + \beta|1\rangle$$

Properties:-

1) $|A\rangle + |B\rangle = |C\rangle = |B\rangle + |A\rangle$

2) Associative

$$|A\rangle + (|B\rangle + |C\rangle) = (|A\rangle + |B\rangle) + |C\rangle$$

3) $|A\rangle = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$

$$\langle A| = (A_1^* \quad A_2^*)$$

* Norm of $|\psi\rangle$

$$\text{Norm}(|\psi\rangle) = \| |\psi\rangle \|$$

$$= \sqrt{\langle \psi | \psi \rangle}$$

if $\langle \psi_1 | \psi_2 \rangle = 0$

↳ ψ_1 and ψ_2 are Called orthonomal basis.

Exa -

$$\| |0\rangle \| = \sqrt{\langle 0|0\rangle} = 1$$

$$\langle 0|0\rangle = (1, 0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

* multiple qubits :-

• Tensor product :-

Let $u_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $u_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$u_1 \otimes u_2 = \begin{bmatrix} x_1 \cdot u_2 \\ y_1 \cdot u_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \\ y_1 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} x_1 x_2 \\ x_1 y_2 \\ y_1 x_2 \\ y_1 y_2 \end{bmatrix}$$

$\Rightarrow |\psi\rangle \otimes |\phi\rangle = |\psi\phi\rangle$

$$|00\rangle = |0\rangle \otimes |0\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Exa:- $|\psi\rangle = i|0\rangle + 7|1\rangle$

$|\phi\rangle = |00\rangle + 3|10\rangle + 7|11\rangle$

$|\psi\phi\rangle = i|000\rangle + 3i|010\rangle + 7i|011\rangle +$

$7|100\rangle + 21|110\rangle + 49|111\rangle$

for single qubit:-

Basis = $\{|0\rangle, |1\rangle\}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$V \in C^1$$

for double qubit :-

Basis = $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$V \in C^2$$

* Ket- Brac Notation :- (outer product)

$$|\alpha\rangle\langle\beta| = \underbrace{\underbrace{|\alpha}_{m\times 1}\; \underbrace{\rangle\langle}_{}\;\underbrace{\beta|}_{1\times n}}_{m\times n}$$

Exa :-

$|00\rangle\langle 10|$

$= |00\rangle\langle 10|$

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

* Bra - Ket Notation:-

$\langle\alpha|\beta\rangle$

= Inner produt

Exa. $\langle 0|1\rangle = (1\;\;0)\begin{pmatrix}0\\1\end{pmatrix} = 0$

Kronecker delta fn :-

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

$\langle\alpha|\beta\rangle = 0$ (states are orthogonal)

$\langle\alpha|\beta\rangle = 1$ (states are identical)

Exa :- $|\alpha\rangle = 2|0\rangle + 7|1\rangle$

$|\beta\rangle = 3|0\rangle + |1\rangle$

$\langle\alpha|\beta\rangle = ?$

$\hookrightarrow$ $\langle\alpha| = 2\langle 0| + 7\langle 1|$

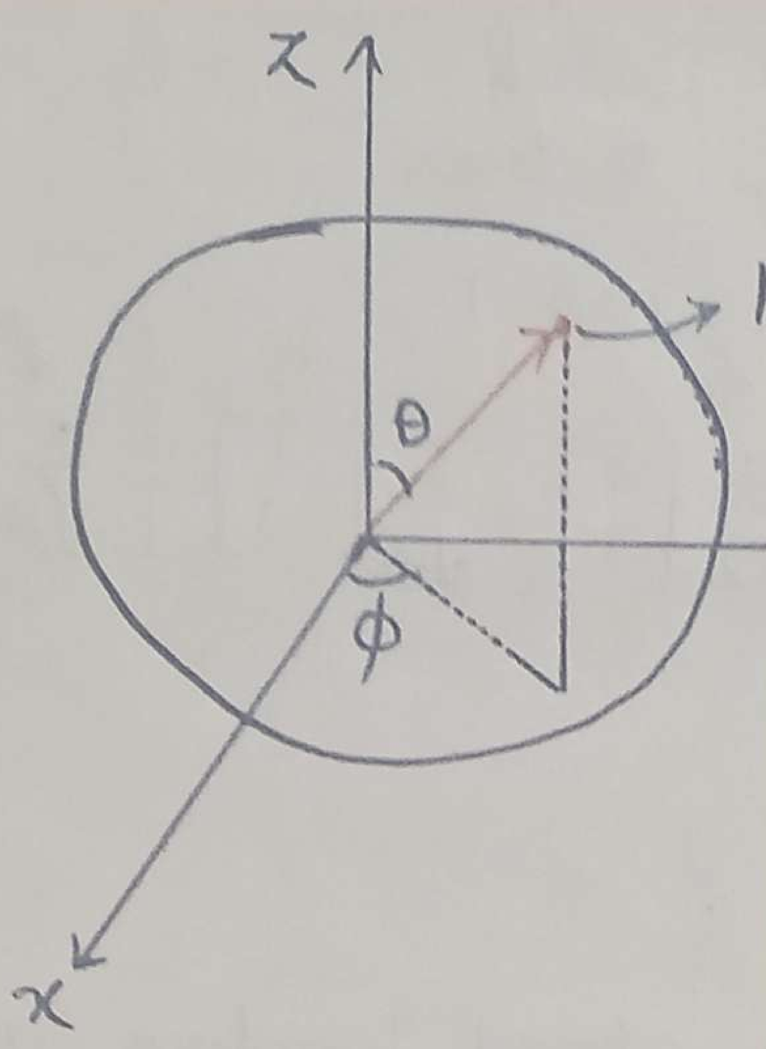
$|\beta\rangle = 3|0\rangle + |1\rangle$

$$\langle\alpha|\beta\rangle = (2\langle 0| + 7\langle 1|)(3|0\rangle + |1\rangle)$$
$$= 6\langle 0|0\rangle + 2\langle 0|1\rangle + 21\langle 1|0\rangle + 7\langle 1|1\rangle$$
$$= 6(1) + 0 + 0 + 7(1)$$
$$= \underline{13}$$

$\Rightarrow$ Real : $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle$

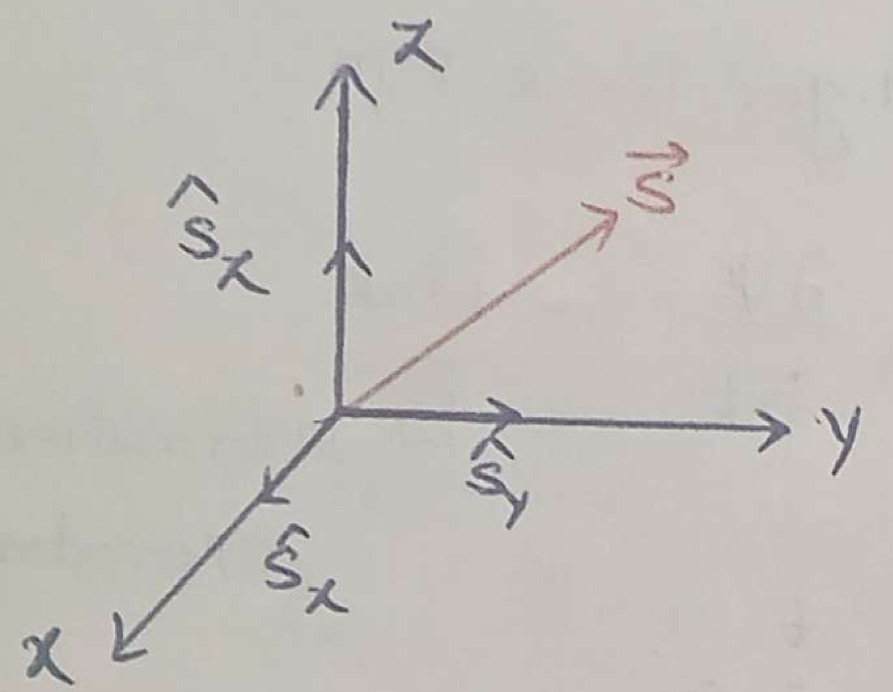
Complex : $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^*$

* Qubit representation on Bloch Sphere:



$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

· Spin Angular momentum:-



$$\hat{S}_x = \frac{\hbar}{2}\sigma_x$$

$$\hat{S}_y = \frac{\hbar}{2}\sigma_y$$

$$\hat{S}_z = \frac{\hbar}{2}\sigma_z$$

Pauli Matrix

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\Downarrow$ E_i: (± 1) $\quad$ $\Downarrow$ ± 1 $\quad$ $\Downarrow$ $+1$

↳ $|A - \lambda I| = 0$

$$\hat{S}_x = \frac{\hbar}{2}(\pm 1), \quad \hat{S}_y = \frac{\hbar}{2}(\pm 1), \quad \hat{S}_z = \frac{\hbar}{2}(\pm 1)$$

$$\hat{S}_x = \pm\frac{\hbar}{2}, \quad \hat{S}_y = \pm\frac{\hbar}{2}, \quad \hat{S}_z = \pm\frac{\hbar}{2}$$

$\lambda = \pm\frac{\hbar}{2}$ [for all three axis)

$\Rightarrow \vec{S}\cdot\hat{n} - \pm\frac{\hbar}{2}$

$\hat{n}$ = Arbitory unit vector

Eigen Valve of S_x (Diagonal Bases)

$$\hat{A}\psi = \lambda\psi$$

$$\because \lambda = \pm\frac{\hbar}{2}$$

$$\hat{S}_x\psi = \lambda\psi$$

$(\lambda = +\frac{\hbar}{2})$

$$\cancel{\frac{\hbar}{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \cancel{\frac{\hbar}{2}}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow y = x$$

$$\psi = \begin{bmatrix} x \\ x \end{bmatrix} = x\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$x = 1/\sqrt{2}$ $\quad (\sqrt{\psi\psi^*} = 1)$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ ↳ diagonal basis

$(\lambda = -\frac{\hbar}{2})$

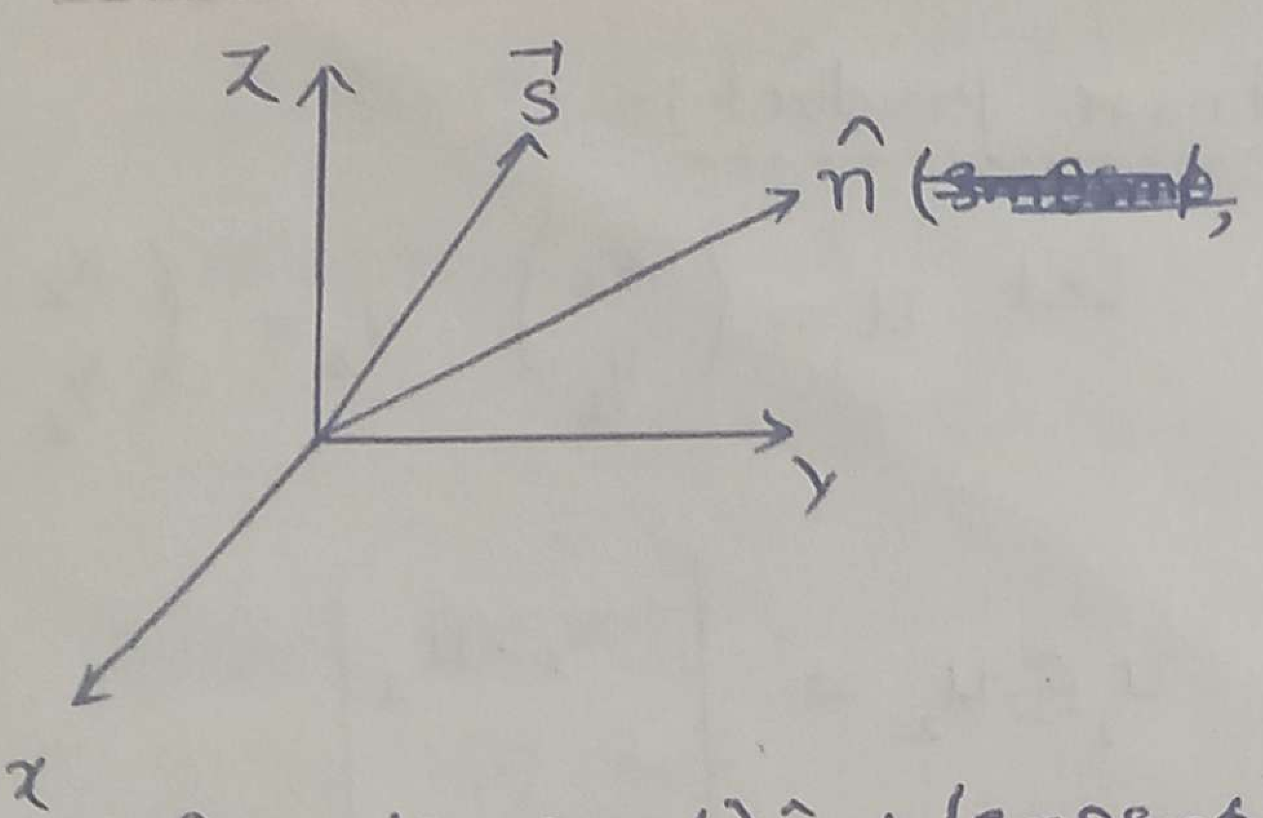
↳ $$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow y = -x$$

$$\psi = \begin{bmatrix} x \\ -x \end{bmatrix}$$

$$\Rightarrow |-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

In any Arbitary direction :-



$\hat{n} = (\sin\theta\cos\phi)\hat{i} + (\sin\theta\sin\phi)\hat{j} + \cos\theta\hat{k}$

measured spin:- $\vec{S}\cdot\hat{n}$

$\hat{A} = \vec{S}\cdot\hat{n}$

$= S_x\sin\theta\cos\phi + S_y\sin\theta\sin\phi + S_z\cos\theta$

Replacing the value of S_x, S_y, S_z

$$\frac{\hbar}{2}\begin{bmatrix}\cos\theta & \sin\theta(\cos\phi - i\sin\phi)\\ \sin\theta(\cos\phi + i\sin\phi) & -\cos\theta\end{bmatrix}$$

$$= \frac{\hbar}{2}\begin{bmatrix}\cos\theta & e^{-i\phi}\sin\theta\\ e^{i\phi}\sin\theta & \cos\theta\end{bmatrix}$$

$\therefore \lambda = \pm\frac{\hbar}{2}$

Case-1 $\lambda = \frac{\hbar}{2}$

$A\psi = \lambda\psi$

$$\frac{\hbar}{2}\begin{bmatrix}\cos\theta & e^{-i\phi}\sin\theta\\ e^{i\phi}\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix}x\\ y\end{bmatrix}$$

$x\cos\theta + ye^{-i\phi}\sin\theta = x$

$ye^{-i\phi}\sin\theta = x(1-\cos\theta)$

$\Rightarrow y\cdot e^{-i\phi}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = x\cdot 2\sin^2\frac{\theta}{2}$

$\Rightarrow y\cdot e^{-i\phi}\cdot\cos\frac{\theta}{2} = x\cdot\sin\frac{\theta}{2}$

$y = x\cdot e^{i\phi}\cdot\tan\frac{\theta}{2}$

$$\psi = \begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}x\\ x e^{i\phi}\tan\theta/2\end{bmatrix}$$

$$\psi = x\begin{bmatrix}\cos\theta/2\\ e^{i\phi}\sin\theta/2\end{bmatrix}$$

$\psi = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

$\psi = \cos\frac{\theta}{2}|0\rangle \pm e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

We take (+)

$$\boxed{\psi = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle}$$

$\psi(-) = \sin\frac{\theta}{2}|0\rangle - e^{i\phi}\cos\frac{\theta}{2}|1\rangle$

→ i) on the x-axis:-

$x_+ : \theta = \frac{\pi}{2},\ \phi = 0$

$= \cos\left(\frac{\pi}{4}\right)|0\rangle + \sin\left(\frac{\pi}{4}\right)|1\rangle$

$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\equiv |+\rangle = H|0\rangle$

$x_- : \theta = \frac{\pi}{2},\ \phi = \pi$

$= \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$\equiv |-\rangle = H|1\rangle$

→ on the y-axis :-

$y_+ : \theta = \frac{\pi}{2}, \phi = \frac{\pi}{2}$

$= \cos\left(\frac{\pi}{4}\right)|0\rangle + e^{i\pi/2}\sin\left(\frac{\pi}{4}\right)|1\rangle$

$= \frac{1}{\sqrt{2}}|0\rangle + i\frac{1}{\sqrt{2}}|1\rangle$

$= \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$y_- : \theta = \frac{\pi}{2}, \phi = \frac{3\pi}{2}$

$= \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

→ on the Z-axis :-

$Z_+ : \theta = 0, \phi = 0$

$= \cos(0)|0\rangle + \sin(0)|1\rangle$

$= |0\rangle$

$Z_- : \theta = \pi, \phi = 0$

$= |1\rangle$

Note - We Can Write -

$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$= \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle$

$\because |e^{i\theta}|^2 = 1$

• Phase Estimation :-

Let Basis $= \{|+\rangle, |-\rangle\}$

$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$

$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

$\because |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle$

$= \frac{1}{2}(|+\rangle + |-\rangle) + \frac{e^{i\theta}}{2}(|+\rangle - |-\rangle)$

$|\psi\rangle = \frac{1+e^{i\theta}}{2}|+\rangle + \frac{1-e^{i\theta}}{2}|-\rangle$

$\because e^{i\theta} = \cos\theta + i\sin\theta$

$\rightarrow P(|+\rangle) = \left|\frac{1+e^{i\theta}}{2}\right|^2$

$= \left|\frac{1+\cos\theta + i\sin\theta}{2}\right|^2$

$= \frac{1}{4}(1+\cos\theta + i\sin\theta)(1+\cos\theta - i\sin\theta)$

$= \frac{1}{4}(1 + 2\cos\theta + \cos^2\theta + \sin^2\theta)$

$= \frac{1}{4}(2 + 2\cos\theta)$

$= \frac{1}{2}(1+\cos\theta)$

$= \frac{1}{2} 2\cos^2\frac{\theta}{2}$

$$\boxed{\begin{aligned} P(|+\rangle) &= \cos^2\theta/_2 \\ P(|-\rangle) &= \sin^2\theta/_2 \end{aligned}}$$

* Postulates of Quantum Computation:-

→ These describe the fundamental principles of quantum mechanics:

1) Representation of Quantum system.
2) How the system changes with respect to time.
3) measurement on quantum system.
4) Composite system

Postulate 1:

→ This defines the stage on which quantum physics plays.

→ Every isolated system is described mathematically by a Hilbert space.

→ The state of system is a vector. (inside the Hilbert space).

→ Any quantum state of the qubit can then be written as a linear combination. (superposition)

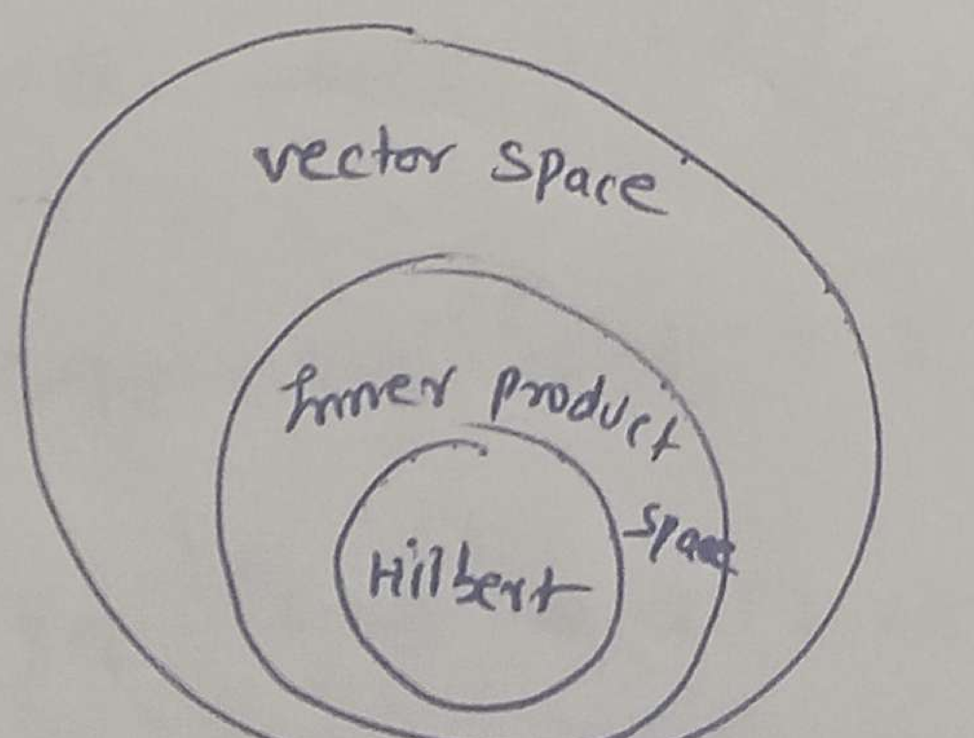
$$|\psi\rangle = \alpha|0\rangle + B|1\rangle$$

$$\alpha, B \in C$$

Normalizing Cond -

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\int_{-\infty}^{\infty} \psi^* \psi \, dz = 1 \checkmark$$



Basis

- Z Basis: $|0\rangle, |1\rangle$
- X-Basis: $\{|+\rangle, |-\rangle\}$: $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
- Y-basis: $|\circlearrowright\rangle, |\circlearrowleft\rangle$: $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

• Postulate - 2 :

→ for the closed quantum system, This describes how the system varying wrt time.

Using shrodinger wave eqn -

$$i\hbar \frac{\partial \psi}{\partial t} = \underbrace{\hat{H}}_{\text{Hermition operator}}\psi$$

$$\Rightarrow \int_{\psi(0)}^{\psi(t)} \frac{\partial \psi}{\psi} = \int_0^t \frac{\hat{H}}{i\hbar} dt$$

$$\Rightarrow [\ln \psi]_{\psi(0)}^{\psi(t)} = \frac{-i\hat{H}}{\hbar} t$$

$$\Rightarrow \ln(\psi(t)) - \ln(\psi(0)) = \frac{-i\hat{H}}{\hbar} t$$

$$\Rightarrow \ln\left[\frac{\psi(t)}{\psi(0)}\right] = \frac{-i\hat{H}}{\hbar} t$$

$$\Rightarrow \boxed{\psi(t) = \psi(0) \cdot \underbrace{e^{-\frac{i\hat{H}}{\hbar}t}}_{\text{Unitary opr.}}}$$

$$|\psi(t)\rangle = e^{-\frac{i\hat{H}t}{\hbar}} |\psi(0)\rangle$$

$$\psi(t) = U \cdot \psi(0)$$

$$UU^{\dagger} = I$$

$$\Rightarrow \psi(t) = U\psi(t_0)$$

$$u = e^{\frac{-i\hat{H}(t-t_0)}{\hbar}}$$

$$\hat{H} = \sum_E E|E\rangle\langle E|$$

E : Eigen value E is an Energy level.

$|E\rangle$: Eigenvector $|E\rangle$ is an Energy Eigenstate.

· ~~Postulate - 3~~ : ~~(measurement)~~

Note :-

A) Hermition operator :.

→ A Linear operator A on a Hilbert space is Hermition (or self adjoint) if it equals its own Conjugate transpose.

$$A^{\dagger} = A$$

→ A is Hermition if $A_{ij} = A_{ji}^*$

→ Properties:-

1) $A|\psi\rangle = \lambda|\psi\rangle \Rightarrow \lambda \in R$

[$|\psi\rangle$: Eigen vector
λ : Eigen value]

2) Eigen vectors Corresponding to different Eigen values-

$$\langle\psi_i|\psi_j\rangle = 0, \text{ if } \lambda_i \neq \lambda_j$$

3)

$$A = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|$$

4) $\langle\phi|A|\phi\rangle \in R$

Exa :- $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$Z^{\dagger} = Z$$

Eigen values $= 1, -1 \in R$

Eigen vectors $= \underbrace{|0\rangle, |1\rangle}_{\text{orthonormal}}$

B) Projection operator

$P_u = |u\rangle\langle u|$ ↳ unit operator $\quad P_u^2 = P$

→ Projects any vector $|\psi\rangle$ onto the direction of $|u\rangle$

$$P_u|\psi\rangle = |u\rangle\langle u|\psi\rangle$$

→ Properties :-

1) $P_u^{\dagger} = (|u\rangle\langle u|)^{\dagger} = |u\rangle\langle u| = P_u^{\dagger}$

$$P_u^{\dagger} = P_u$$

$$(|\psi_1\rangle\langle\psi_2|)^{\dagger} = (|\psi_2\rangle\langle\psi_1|)$$

2) $P_u^2 = P_u \cdot P_u$

$$(|u\rangle\underbrace{\langle u|)(|u\rangle}_{=1}\langle u|)$$

$$= |u\rangle\langle u|$$

3) $\sum_u P_u = I$

• Postulate - 3 (Measurement):

→ Any ~~phy~~ measurable physical quantity (observable) is represented by a Hermition operator M.

1) measurement operator : projection opr.

↳ Represented by M_m

$$M_m = |m\rangle\langle m|$$

if $\psi_i = \alpha|0\rangle + \beta|1\rangle$

$$M_0 \psi_i = |0\rangle\langle 0| (\alpha|0\rangle + \beta|1\rangle)$$

$$= |0\rangle[\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle]$$

$$= \underline{\alpha|0\rangle}$$

$$M_1 \psi_i = \beta|1\rangle$$

2) Probability :-

$$\underline{P_m = \langle\psi|M_m|\psi\rangle}$$

$\because M_m^{+} = M_m \quad , \quad M_m^2 = M_m \cdot M_m$

$$= M_m$$

$$= M_m^{+} M_m$$

$$P_m = \langle\psi| M_m^2 |\psi\rangle$$

$$P_m = \langle\psi| M_m^{+} M_m |\psi\rangle$$

∵ Completeness Rule -

$$\sum_\psi |\psi\rangle\langle\psi| = I$$

$$\sum_m M_m = I$$

$$= \sum_m M_m^{+} M_m = I$$

3) Post measurement state

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^{+} M_m|\psi\rangle}}$$

Exa- $\quad \psi = \alpha|0\rangle + \beta|1\rangle$

Post measurement of $|0\rangle$

$$\frac{M_0|\psi\rangle}{\sqrt{\langle\psi|M_0|\psi\rangle}} = \frac{\alpha|0\rangle}{\sqrt{|\alpha|^2}}$$

$$= \frac{\alpha}{|\alpha|}|0\rangle$$

$$\boxed{\frac{\alpha}{|\alpha|} = \text{Global Phase}}$$

$$= e^{i\theta}|0\rangle$$

Post measurement of $|1\rangle$

$$= \frac{\beta|1\rangle}{|\beta|} = e^{i\theta}|1\rangle$$

for x- Basis:-

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\psi = \alpha|0\rangle + \beta|1\rangle$$

$$= \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|-\rangle$$

• $M_+ = |+\rangle\langle +|$

$$M_+|\psi\rangle = |+\rangle\langle +|\psi\rangle$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \alpha+\beta/\sqrt{2} \\ \alpha-\beta/\sqrt{2} \end{pmatrix}$$

$$= \frac{\alpha+\beta}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2) $P_+ = \langle \psi | M_+ | \psi \rangle$

$$= \begin{bmatrix} \frac{(\alpha+\beta)^*}{\sqrt{2}} & \frac{(\alpha-\beta)^*}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\alpha+\beta}{\sqrt{2}} \\ \frac{\alpha-\beta}{\sqrt{2}} \end{bmatrix}$$

$$= \frac{|\alpha+\beta|^2}{2}$$

3) Post measurement : $\frac{M_+ |\psi\rangle}{\sqrt{P_+}}$

$$= \frac{\left(\frac{\alpha+\beta}{\sqrt{2}}\right) |+\rangle}{\left|\frac{\alpha+\beta}{\sqrt{2}}\right|} = e^{i\theta} |+\rangle$$