Digital Communication System

Advantages –

1. Noise Interference is less
2. Encoding and Decoding consist Parity bits to detect and correct error.
3. High Processing Capacity
4. Multiplexing would be easy
5. Simple computation

DisAdvantages –

1. High Bandwidth Required
2. Costly
3. Complex Transmitter and Receiver
4. Synchronization is required between Tx and Rx
5. Can not work upon Traditional Tx and Rx

Baseband vs Passband

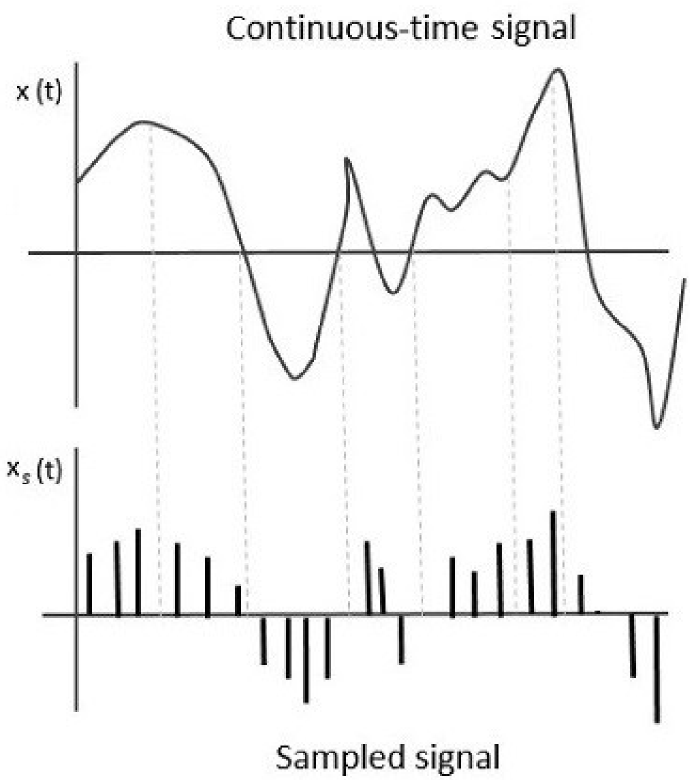
Feature	Baseband	Passband
Signal Type	Digital (original)	Analog (modulated digital)
Channel Type	Low-pass	Band-pass
Transmission Distance	Short	Long
Uses	Ethernet, LAN	Wireless, WAN, modem systems
Modulation Needed	No	Yes
Bandwidth Usage	Entire bandwidth	Specific frequency band

Sampling Theorem –

Real World Analog Signals are converted into digital signal so that they can be processed by the computer .

The process of converting continuous time signals into equivalent discrete time signals, can be termed as **Sampling**. A certain instant of data is continually sampled in the sampling process.

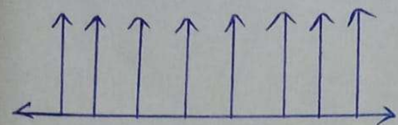
The following figure shows a continuous-time signal **x(t)** and the corresponding sampled signal **x_s(t)**. When **x(t)** is multiplied by a periodic impulse train, the sampled signal **x_s(t)** is obtained.



A **sampling signal** is a periodic train of pulses, having unit amplitude, sampled at equal intervals of time Ts, which is called as **sampling time**.

$$fs = \frac{1}{Ts}$$

Where f_s = Sampling Frequency



Impulse train- $\sum \delta(t-nT_s)$

$$x_s(t) = x(t) \cdot \sum \delta(t-nT_s)$$

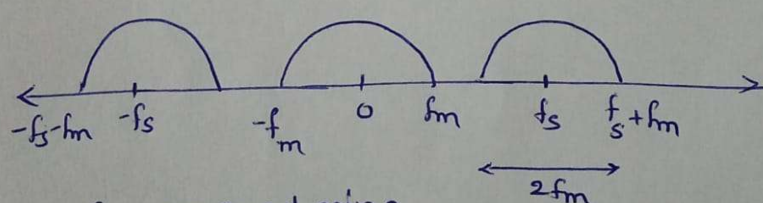
$$\because \quad x(t) \xleftrightarrow{FT} X(f)$$

$$\sum \delta(t-nT_s) \longleftrightarrow f_s \sum \delta(f-nf_s)$$

$$X_s(f) = X(f) * f_s \sum \delta(f-nf_s)$$

$$= f_s \sum X(f) * \delta(f-nf_s)$$

$$= f_s \sum X(f-nf_s)$$



for No overlapping

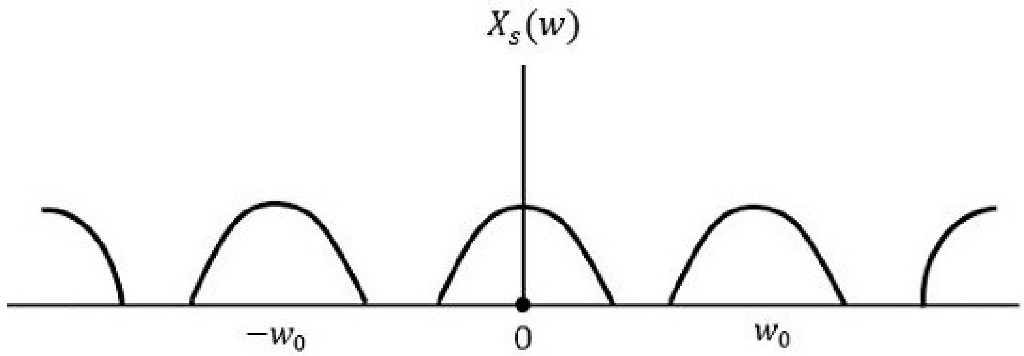
$$f_s \geq 2f_m$$

where f_m – max freq. of $X(f)$

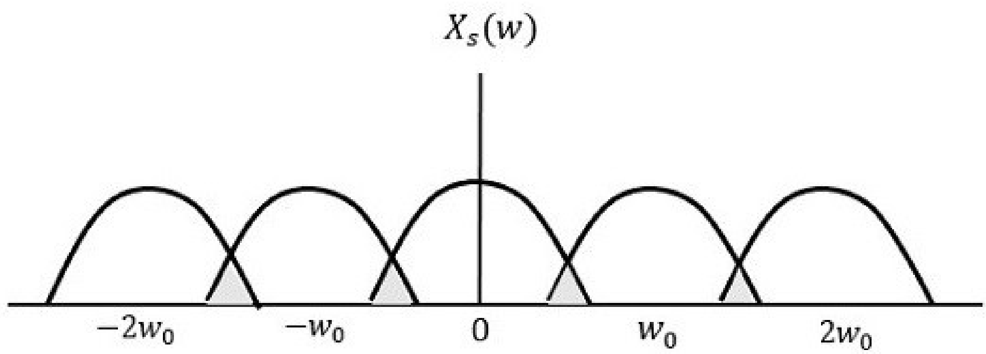
If the sampling rate is equal to twice the maximum frequency of the given signal f_m, then it is called as **Nyquist rate**.

The sampling theorem, which is also called as **Nyquist theorem**, delivers the theory of sufficient sample rate in terms of bandwidth for the class of functions that are bandlimited.

If the signal is sampled above Nyquist rate, then the original signal can be recovered.

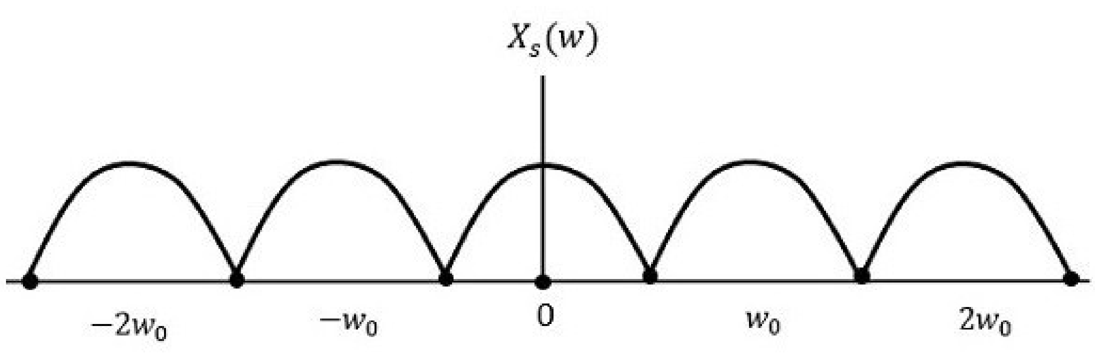


If the same signal is sampled at a rate less than **2**f_m, then the sampled signal would look like the following figure.



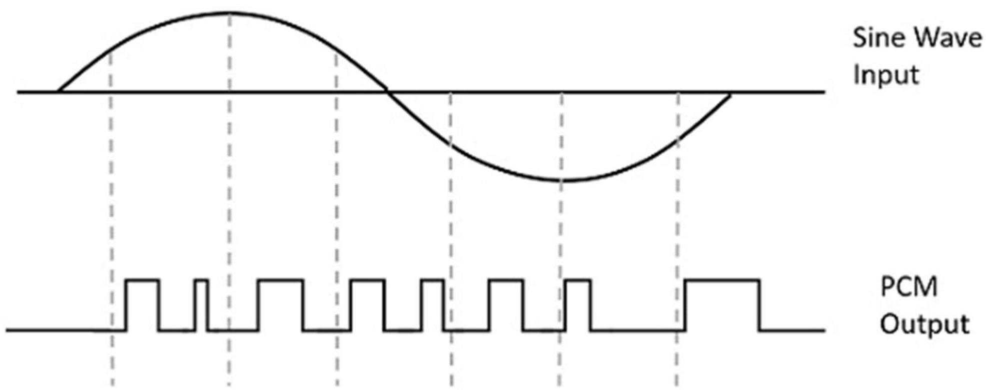
We can observe from the above pattern that there is over-lapping of information, which leads to mixing up and loss of information. This unwanted phenomenon of over-lapping is called as **Aliasing**.

Hence, the sampling rate of the signal is chosen to be as Nyquist rate.

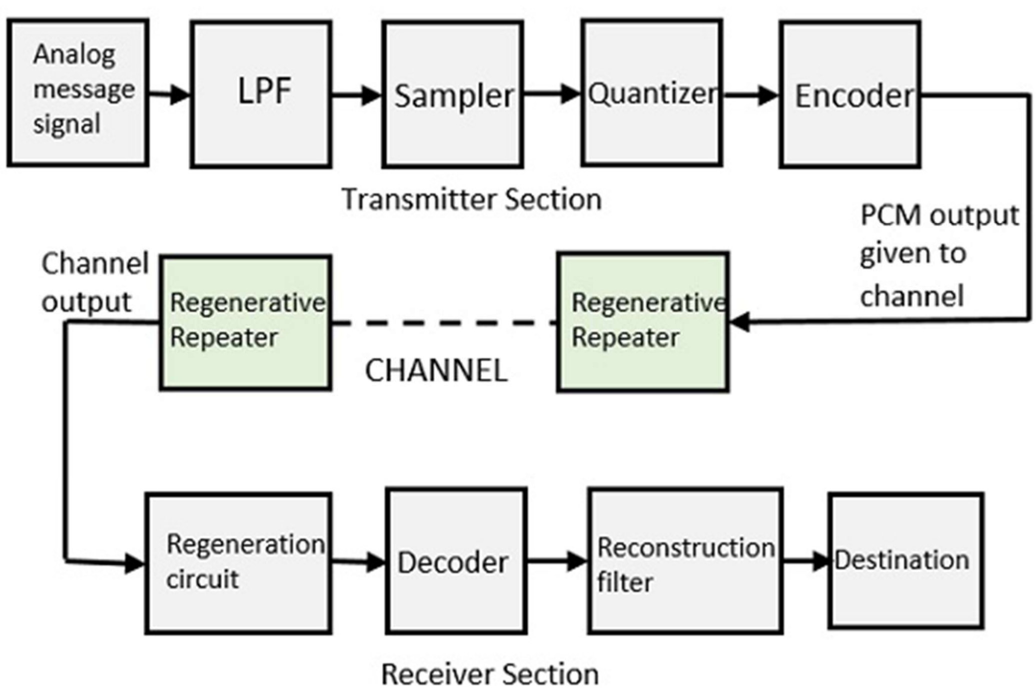


Pulse Code Modulation -

A signal is pulse code modulated to convert its analog information into a binary sequence, i.e., **1s** and **0s**. The output of a PCM will resemble a binary sequence.



Basic Elements of PCM –



Low Pass Filter-

This filter eliminates the high frequency components present in the input analog signal which is greater than the highest frequency of the message signal, to avoid aliasing of the message signal.

Sampler –

This is the technique which helps to collect the sample data at instantaneous values of message signal, so as to reconstruct the original signal

Quantizer -

Quantizing is a process of reducing the excessive bits and confining the data. The sampled output when given to Quantizer, reduces the redundant bits and compresses the value.

Encoder -

The digitization of analog signal is done by the encoder. It designates each quantized level by a binary code. The sampling done here is the sample-and-hold process. These three sections (LPF, Sampler, and Quantizer) will act as an analog to digital converter. Encoding minimizes the bandwidth used.

Regenerative Repeater –

This section increases the signal strength. The output of the channel also has one regenerative repeater circuit, to compensate the signal loss and reconstruct the signal, and also to increase its strength.

Decoder -

The decoder circuit decodes the pulse coded waveform to reproduce the original signal. This circuit acts as the demodulator.

Reconstruction Filter -

After the digital-to-analog conversion is done by the regenerative circuit and the decoder, a low-pass filter is employed, called as the reconstruction filter to get back the original signal. Hence, the Pulse Code Modulator circuit digitizes the given analog signal, codes it and samples it, and then transmits it in an analog form. This whole process is repeated in a reverse pattern to obtain the original signal.

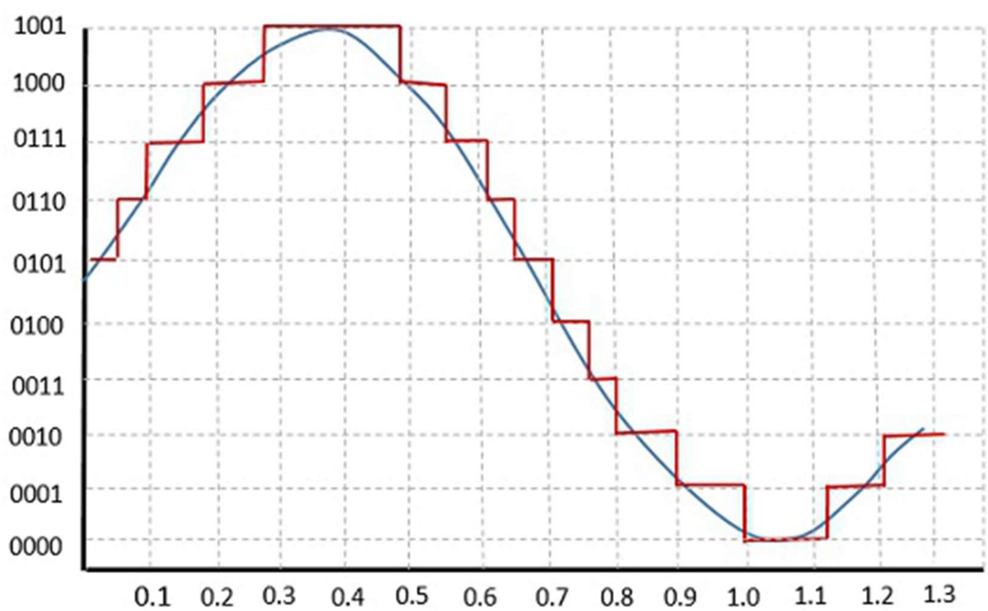
Quantization –

- Quantization is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.
- Both sampling and quantization result in the loss of information.
- The quality of a Quantizer output depends upon the number of quantization levels used.
- The discrete amplitudes of the quantized output are called as representation levels or reconstruction levels.
- The spacing between the two adjacent representation levels is called a quantum or step-size.

$$\text{step size} = V_{max} – V_{min} / L$$

- If we have to represent result in n bits Then number of levels are -

$$\text{Number of Levels} = 2^n$$



There are two types of Quantization

- Uniform Quantization (Step size is Fixed)
- Non-uniform Quantization. (Step Size is Variable)

1. <u>Uniform Quantization :</u>

- There are two types of uniform quantization.
  1) Mid – Rise type (Even Number of Levels)
  2) Mid – Tread type (Odd Number of Levels)

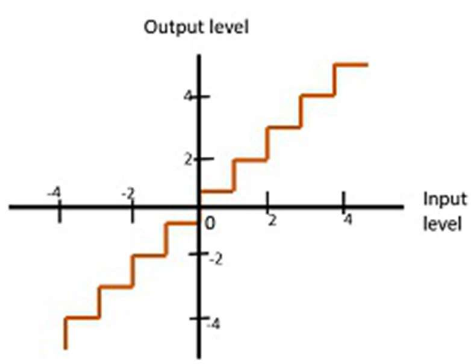


Fig 1 : Mid-Rise type Uniform Quantization

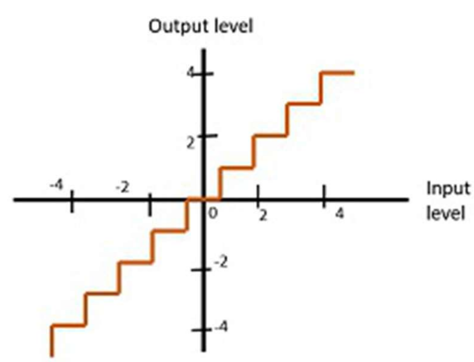


Fig 2 : Mid-Tread type Uniform Quantization

<u>Quantization Error :</u>

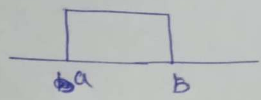
$$\text{Quantization Error } q_e = x_q(nT_s) - x(nT_s)$$

$$\text{Step size } \Delta = 2x_{max} / L$$

$$(q_e)_{max} = \left|\frac{\Delta}{2}\right|$$

This is a Uniformly Distributed Function in – Δ / 2 to Δ / 2

∵ P.d.f of Uniformly distributed fn –



$$f_x(x) = \frac{1}{b-a}$$

if $b = +\frac{\Delta}{2}$, $a = -\frac{\Delta}{2}$

$$f_x(x) = \frac{1}{\frac{\Delta}{2} - \left(-\frac{\Delta}{2}\right)} = \frac{1}{\Delta}$$

→ x is any Random signal with Power P.

• Noise Power –

$$N_q = E(\varepsilon^2)$$

$$= \int_{-\alpha}^{\alpha} \varepsilon^2 . f_x(x)\, d\varepsilon$$

$$= \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 . \frac{1}{\Delta}\, d\varepsilon$$

$$\boxed{N_q = \frac{\Delta^2}{12}}$$ (mean square value)

* Signal to the Noise Ration:-

$$SNR = \frac{S}{N_q} = \frac{\text{Avg. signal power}}{\text{Avg. Quantization Power}}$$

$$SNR = \frac{P}{\Delta^2/12}$$

$$= \frac{12P}{\Delta^2} = \frac{12P\, 2^{2n}}{4A^2} = \frac{3P2^{2n}}{A^2}$$

$$\boxed{SNR = \frac{3PL^2}{A^2}}$$

P – Normalised Power ≤ 1

$$\Rightarrow SNR \leq \frac{3L^2}{1}$$

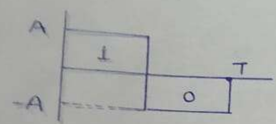
$$\boxed{SNR \leq 3L^2 \approx 3 \times 2^{2n}}$$

In decibals – $\left(\frac{S}{N_q}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N_q}\right)$

$$\leq 10 \log_{10}(3 \times 2^{2n})$$

$$\leq (4.8 + 6n)\ dB$$

Case-1 Binary Bit -



$$P = \frac{1}{T}\left[A^2 \times \frac{T}{2} + (-A)^2 \times \frac{T}{2}\right]$$

$$P = A^2$$

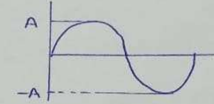
$$SQNR = SNR = \frac{3PL^2}{A^2} = \frac{3A^2L^2}{A^2}$$

$$= 3L^2 = 3 \times 2^{2n}$$

$$SQNR\Big|_{dB} = 10 \log(3 \times 2^{2n})$$

$$= 4.8 + 6n$$

Case-2 Sinusoidal signal



$$\because P = \frac{[A^2_{Rms}]}{R}$$

$$\because A_{Rms} = \frac{A}{\sqrt{2}}$$

$$R = 1$$

$$P = \frac{A^2}{2}$$

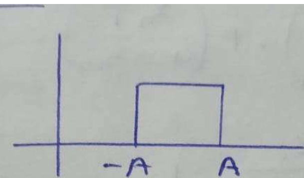
$$SQNR = \frac{3 \times \frac{A^2}{2} \times L^2}{A^2} = 1.5 \times 2^{2n}$$

$$SQNR\Big|_{dB} = 10 \log(1.5 \times 2^{2n})$$

$$= 1.76 + 6n$$

$$\approx 1.8 + 6n$$

$$\text{Crest factor} = \frac{A}{\sqrt{P}}$$



$$f_x(x) = \frac{1}{2A}$$

$$P = \int_{-\infty}^{\infty} x^2 \cdot f_x(x)\, dx$$

$$P = \int_{-A}^{A} \frac{1}{2A} x^2 dx$$

$$P = \frac{1}{2A} \times \frac{2A^3}{3} = \frac{A^2}{3}$$

$$SQNR = L^2 = 2^{2n}$$

Transmission bandwidth in Pulse Code Modulation

The transmission **bandwidth of a PCM system is associated with a number of bits per sample**.

Let us consider each quantizer level is represented by ‘n’ binary digits. Then the levels represented by n binary digits is given as,

$$\mathbf{L = 2^n}$$

: q is the digital level of the quantizer

Every sample is changed into n bits, thus, a number of bit per sample is ‘n’.

As we have already discussed the number of samples per second is f_s. Hence the number of bits per second which is also termed as signalling rate is given as,

$$\mathbf{r = n\, f_s}$$

As transmission bandwidth is half the signalling rate, hence

$$BW \geq \frac{1}{2} r$$

As we have already discussed, $\mathbf{r = nf_s}$

Therefore,

$$BW \geq \frac{1}{2} n f_s$$

But we know, $\mathbf{f_s \geq 2f_m}$

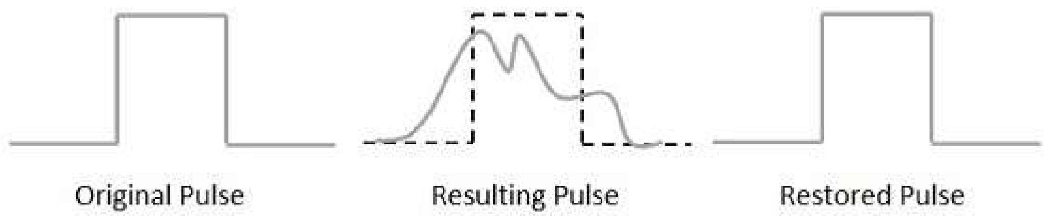
Thus the bandwidth of the PCM system is given as,

$\mathbf{BW \geq n\, f_m}$

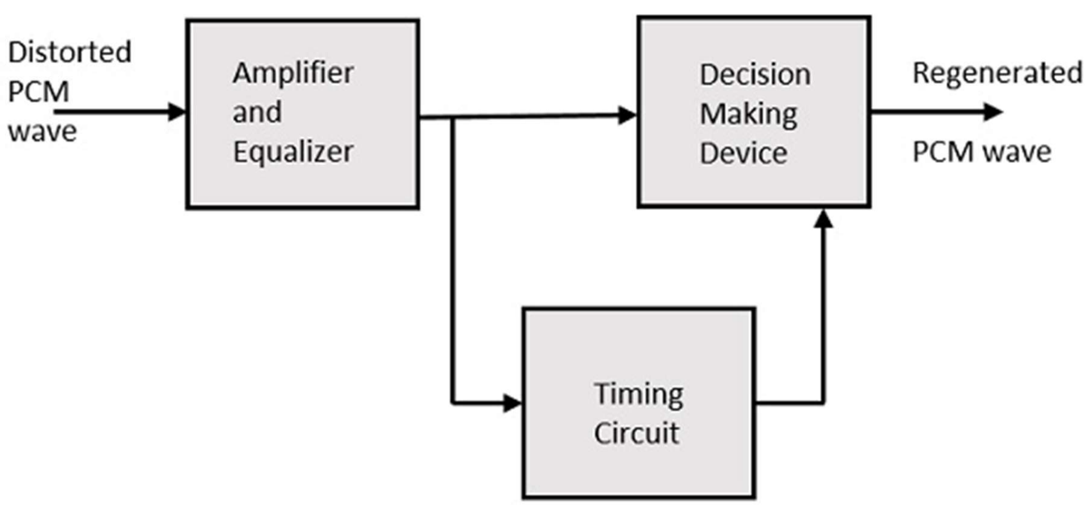
Regenerative Repeaters –

A PCM wave, after transmitting through a channel, gets distorted due to the noise introduced by the channel.

The regenerative pulse compared with the original and received pulse, will be as shown in the following figure.



For a better reproduction of the signal, a circuit called as **regenerative repeater** is employed in the path before the receiver. This helps in restoring the signals from the losses occurred. Following is the diagrammatical representation.



Block diagram of a regenerative repeater

This consists of an equalizer along with an amplifier, a timing circuit, and a decision making device. Their working of each of the components is detailed as follows.

Equalizer

The channel produces amplitude and phase distortions to the signals. This is due to the transmission characteristics of the channel. The Equalizer circuit compensates these losses by shaping the received pulses.

Timing Circuit

To obtain a quality output, the sampling of the pulses should be done where the signal to noise ratio (SNR) is maximum. To achieve this perfect sampling, a periodic pulse train has to be derived from the received pulses, which is done by the timing circuit.

Hence, the timing circuit, allots the timing interval for sampling at high SNR, through the received pulses.

Decision Device

The timing circuit determines the sampling times. The decision device is enabled at these sampling times. The decision device decides its output based on whether the amplitude of the quantized pulse and the noise, exceeds a pre-determined value or not.

Advantages

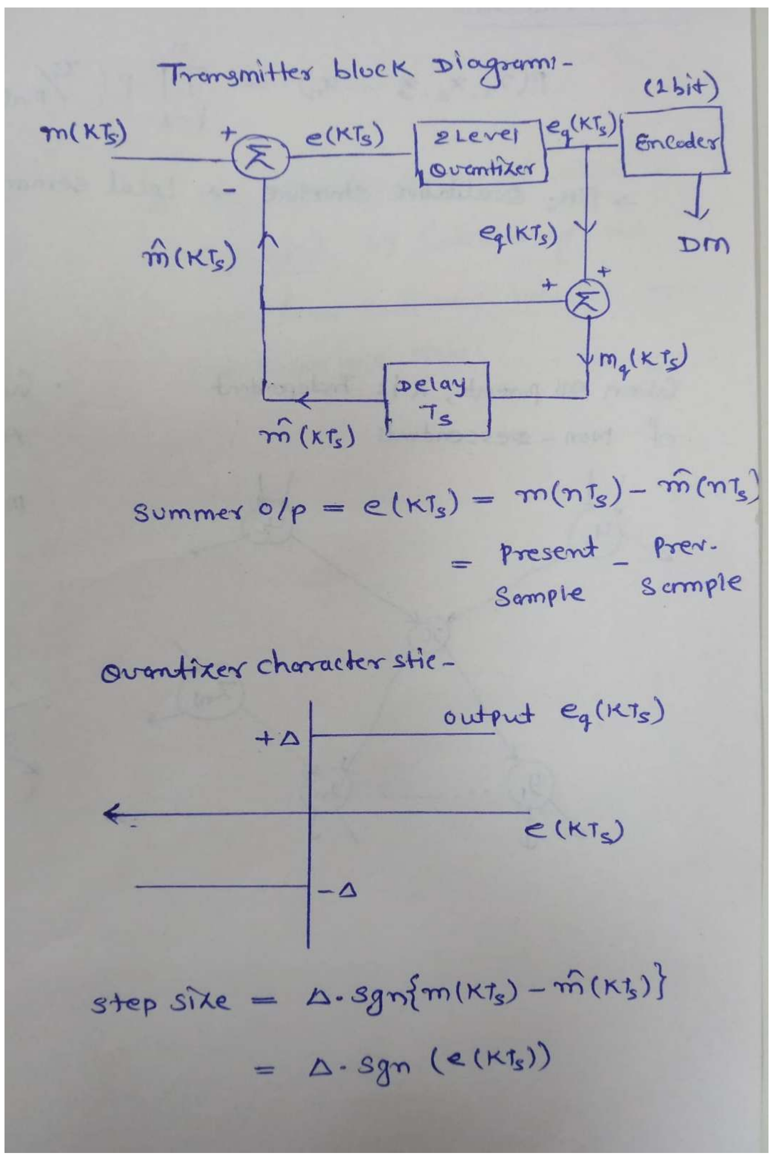
- Pulse Code Modulation is used in long-distance communication.
- The efficiency of the transmitter in PCM is high.
- Higher noise immunity is seen.
- Efficient method.

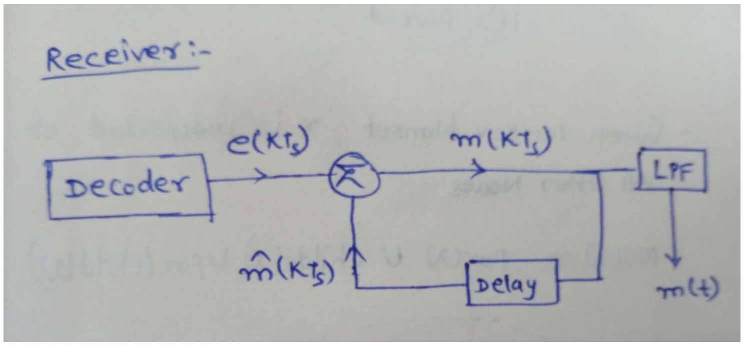
Disadvantages

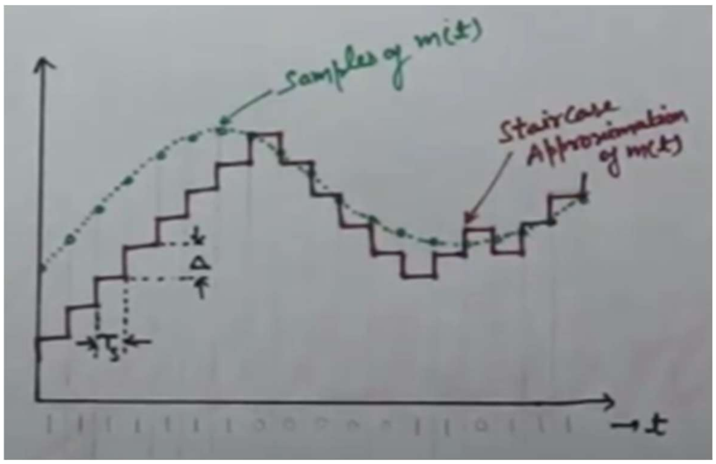
- The bandwidth requirement is high.
- PCM is a complex process, since it involves encoding, decoding and quantisation of the circuit.

Delta Modulation –

- The sampling rate of a signal should be higher than the Nyquist rate, to achieve better sampling.
- The quality is moderate.
- The design of the modulator and the demodulator is simple.
- The stair-case approximation of output waveform.
- The step-size is very small, i.e., **Δ** (delta).
- It uses the simplest possible quantizer that is two level (or one bit) Quantizer.





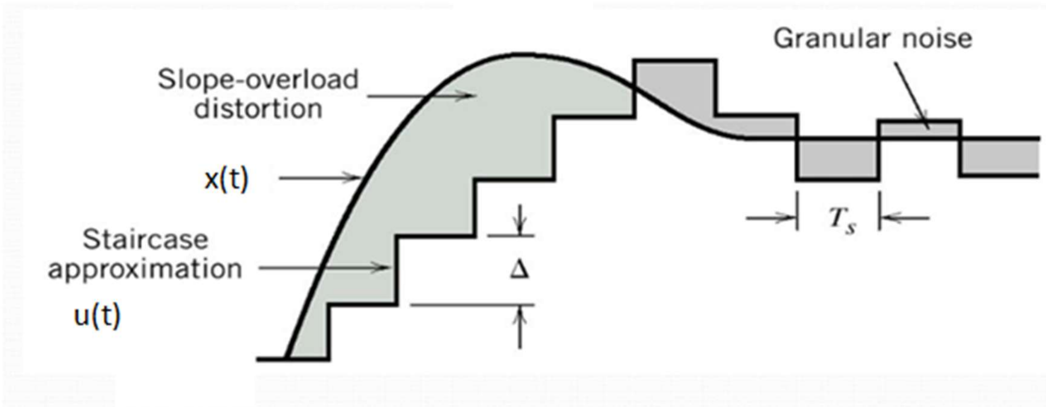


Drawbacks –

1. Slope Overload Distortion
2. Granular Noise or Idle Noise

1. Slope Overload Distortion

- Occurs when the input signal changes rapidly.
- The step size (Δ) is too small to follow the steep slope.
- The staircase approximation lags behind the input signal.
- Results in large error between input signal x(t) and staircase signal u(t).
- Solution: Increase step size when input signal has a high slope.



2. Granular (Idle) Noise

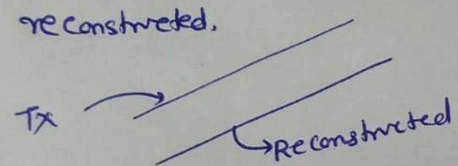
- Occurs when **input signal varies slowly** or is almost flat.
- A **large step size (Δ)** causes the output to oscillate around the input.
- Results in small but **persistent error**.
- Known as **granular noise**.
- **Solution**: Decrease step size during slow signal variation.

→ The value of Δ for which $m(t)$ can be perfectly reconstructed is called Δ_{opt} or optimal step size.

→ $\Delta < \Delta_{opt} \Rightarrow$ Slope overload

→ $\Delta > \Delta_{opt} \Rightarrow$ Granular Noise Error

→ If the Rate of change (i.e. slope) of the re-constructed signal is same as the transmitted signal then transmitted message signal can be perfectly reconstructed.



→ In case of perfect reconstruction –

slope of $m(t)$ = slope of ReCon. signal

$$\frac{d}{dt}[m(t)] = \frac{\Delta_{opt}}{T_s}$$

→ if $\frac{\Delta}{T_s} < \frac{d}{dt}(m(t)) \Rightarrow$ slope overload

→ if $\frac{\Delta}{T_s} > \frac{d}{dt}[m(t)] \Rightarrow$ Granular Noise

SQNR for Delta Modulation -

$\because \; x(t) = A\sin(2\pi f_m t)$

$$\frac{\Delta_{opt}}{T_s} = \frac{d}{dt}[x(t)]\Big|_{max}$$

$$= [A\cdot\cos(2\pi f_m t)\cdot 2\pi f_m]_{max}$$

$$\frac{\Delta_{opt}}{T_s} = 2\pi A f_m$$

$$\Delta_{opt} = \frac{2\pi A f_m}{f_s}$$

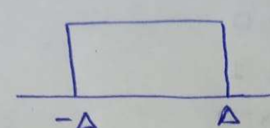
$$\because \; P = \frac{A_{Rms}^2}{R}$$

$(R=1)$

$$P = \frac{A^2}{2}$$

$$P = \frac{1}{2}\left(\frac{f_s \Delta}{2\pi f_m}\right)^2$$

Noise Power



$$f_x(x) = \frac{1}{2\Delta}$$

$$N_q = \langle n_x^2(e)\rangle = \int_{-\infty}^{\infty} f_x(x)\cdot x^2\,dx$$

$$= \int_{-\Delta}^{\Delta} \frac{1}{2\Delta}\cdot x^2\,dx$$

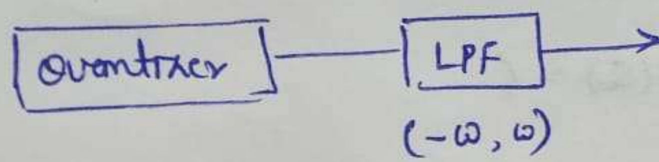
$$= \frac{1}{2\Delta}\left[\frac{x^3}{3}\right]_{-\Delta}^{\Delta}$$

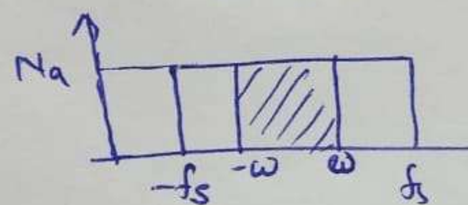
$$= \frac{1}{2\Delta}\left(\frac{2\Delta^3}{3}\right) = \frac{\Delta^2}{3}$$

$$SQNR = \frac{P}{N_q}$$

$$= \frac{3 f_s^2}{8\pi^2 f_m^2}$$

Using LPF





$$N_q = \int_{-\omega}^{\omega} \frac{N_a}{2f_s}\, d\omega$$

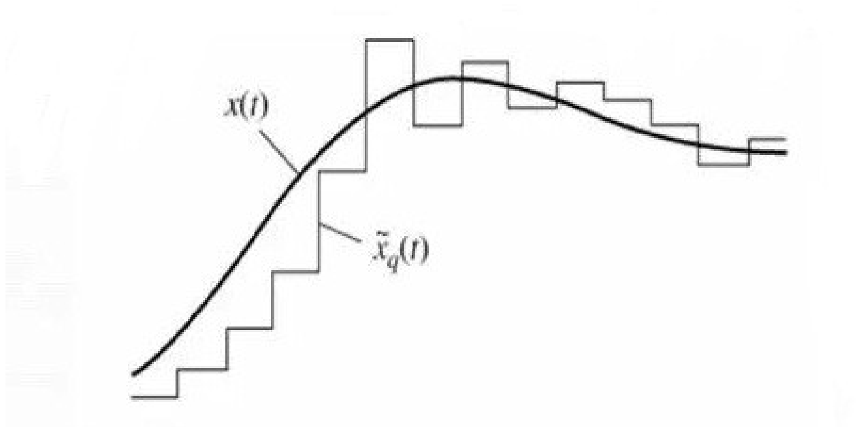
$$N_q = \frac{N_a \omega}{f_s} = \left(\frac{\Delta^2}{3}\right)\frac{\omega}{f_s}$$

$$N_q = \left(\frac{\Delta^2}{3}\right)\frac{f_m}{f_s}$$

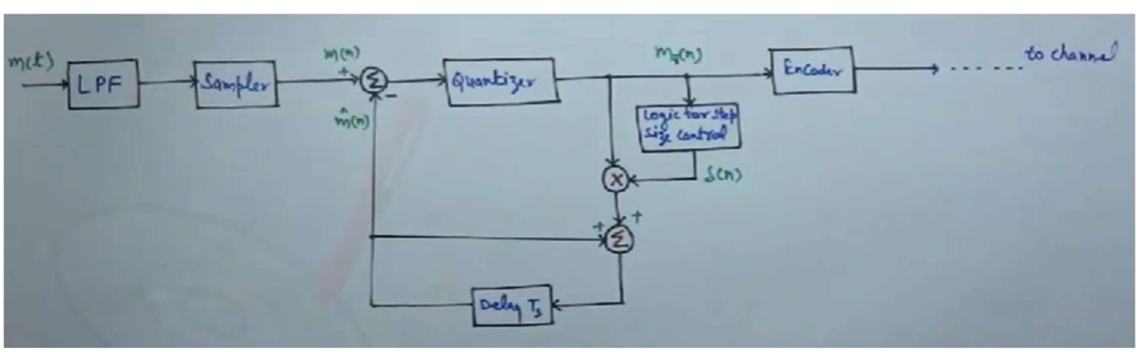
$$SQNR = \frac{3 f_s^3}{8\pi^2 f_m^2 \cdot F_m}$$

Adaptive Delta Modulation

- Modified Version of Delta Modulation
- To Overcome slope overload distortion and Granular Noise in delta modulation , Adaptive delta modulation is used.
- Here the step size Δ of the Quantizer is not a constent but varies with time.



ADM Transmitter -



$\therefore \; \delta_{min} \leq \delta(n) \leq \delta_{max}$

or $\Delta_{min} \leq \Delta(n) \leq \Delta_{max}$

upper Limit Δ_{max} Controls the Amount of slope overload distortion and Lower Limit Δ_{min} Controls the Amount of Granular Noise.

$$\Delta(n) = g(n) \cdot \Delta(n-1)$$

where $g(n)$ is time varying Gain depends on the present output $m_q(n)$ and previous value $m_q(n-1)$, $m_q(n-2)$ _

successive value of $m_q(n)$ and $m_q(n-1)$ are Compared to detect probable slope overload $(m_q(n) = m_q(n-1))$ or Probable granularity $[m_q(n) \neq m_q(n-1)]$

Then $g(n)$ Can be written as-

$$g(n) = \begin{cases} P, & \text{if } m_q(n) = m_q(n-1) \\ 1/P, & \text{if } m_q(n) \neq m_q(n-1) \end{cases}$$

Where $P > 1$

Typically $P_{opt} = 1.5 = 3/2$ for speach signal

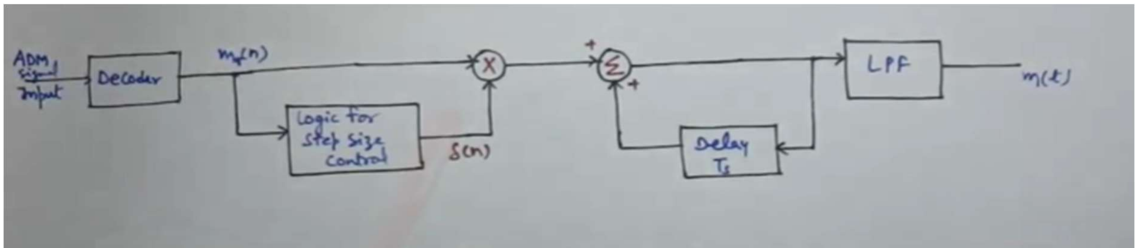


Fig. 3 ADM Receiver