* Communication - System

* Fourier - Transformation :-

↳ This is a mathematical Tool used for frequecy analysis of signals.

↳ This can exist for - (i) Energy signal (ES) (ii) Power signal (PS)

(iii) Impulse related signal.

→ $x(t) \longleftrightarrow \underline{X(j\omega)}$ or $X(f)$

↳ $X(\omega)$

(1) $x(t) \longrightarrow X(\omega)$

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt}$$

(Fourier Transform)

(2) $X(\omega) \longrightarrow x(t)$

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} d\omega}$$

(Inverse Fourier Transform)

• Existence of Fourier Transformation :-

1) The fn may have single value and having finite max and min.
2) The fn has a finite disContinuity in any finite Interval.
3) $\int_{-\infty}^{\infty} f(t)\, dt < \infty$ (Absolutely Integral signal)

↳ These Conditons are sufficient but not Necessary.

* Properties :-

(1) Linearity :-

$x_1(t) \longleftrightarrow X_1(\omega)$

$x_2(t) \longleftrightarrow X_2(\omega)$

$$\boxed{\alpha x_1(t) + \beta x_2(t) \rightleftharpoons \alpha X_1(\omega) + \beta X_2(\omega)}$$

(2) Conjugation :-

$x(t) \longleftrightarrow X(\omega)$

$$\boxed{x^*(t) \longleftrightarrow X^*(-j\omega) / X^*(-\omega)}$$

Proof :-

$x(t) \longleftrightarrow X(j\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$X^*(\omega) = \int_{-\infty}^{\infty} x^*(t) \cdot e^{j\omega t} dt$$

$\omega \longrightarrow (-\omega)$

$$X^*(-\omega) = \int_{-\infty}^{\infty} x^*(t) \cdot e^{-j\omega t} dt$$

$x^*(t) \longleftrightarrow X^*(-\omega)$

(3) Area under $x(t)$ :-

$$\text{Area under } x(t) = \int_{-\infty}^{\infty} x(t)\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

At $\omega = 0$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt$$

$$\boxed{\text{Area under } x(t) = X(\omega)\Big|_{\omega=0}}$$

(4) Area Under $X(\omega)$ :-

$$\text{Area Under } X(\omega) = 2\pi x(t)\Big|_{t=0}$$

(5) Time Reversal:-

$$x(t) \longleftrightarrow X(j\omega)$$

$$x(-t) \longleftrightarrow X(-\omega)$$

Proof - $$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$t \longrightarrow -t$$

$$x(-t) \longleftrightarrow X'(\omega)$$

$$X'(\omega) = \int_{-\infty}^{\infty} x(-t)\cdot e^{-j\omega t}\,dt$$

$$-t = \tau \Rightarrow t = -\tau \Rightarrow dt = -d\tau$$

$$= -\int_{\infty}^{-\infty} x(\tau)\cdot e^{j\omega\tau}\,d\tau$$

$$X'(\omega) = \int_{-\infty}^{\infty} x(\tau)\cdot e^{-j(-\omega)\tau}\,d\tau$$

$$x(-t) \longleftrightarrow X(-\omega)$$

(6) Time-Scaling:-

$$x(t) \longleftrightarrow X(\omega)$$

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \;;\; a \neq 0$$

Proof:- $$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$x(at) \rightleftharpoons X'(\omega)$$

(I) $a > 0 \Rightarrow$ $$X'(\omega) = \int_{-\infty}^{\infty} x(at)\cdot e^{-j\omega t}\,dt$$

$$at = \tau \Rightarrow t = \frac{\tau}{a}$$

$$dt = \frac{1}{a}d\tau$$

$$X'(\omega) = \int_{-\infty}^{\infty} x(\tau)\cdot e^{-j\omega \tau/a}\cdot \frac{d\tau}{a}$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} x(\tau)\cdot e^{-j\left(\frac{\omega}{a}\right)\tau}\,d\tau = \frac{1}{a}X\left(\frac{\omega}{a}\right)$$

(7) Time-shifting :-

$$x(t) \longleftrightarrow X(\omega)$$

$$x(t \pm t_0) \longleftrightarrow X(\omega)e^{\pm j\omega t_0}$$

(8) frequency-shifting :-

$$x(t) \longleftrightarrow X(\omega)$$

$$e^{\pm j\omega_0 t}(x(t)) \longleftrightarrow X(\omega \mp \omega_0)$$

(9) Convolution in time :-

$$x_1(t) \longleftrightarrow X_1(\omega)$$

$$x_2(t) \longleftrightarrow X_2(\omega)$$

$$x_1(t) * x_2(t) \longleftrightarrow X_1(\omega) \times X_2(\omega)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau$$

(10) Multiplication in Time :- $\rightarrow$ (convolution in freq)

$$x_1(t) \longleftrightarrow X_1(\omega)$$

$$x_2(t) \longleftrightarrow X_2(\omega)$$

$$\boxed{x_1(t)\cdot x_2(t) \longleftrightarrow \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]}$$

(11) Diffrentiation in Time :-

$$x(t) \longleftrightarrow X(\omega)$$

$$\frac{d^K}{dt^K}(x(t)) \longleftrightarrow (j\omega)^K\cdot X(\omega)$$

Proof-

(12) Integration in Time :-

$$x(t) \longleftrightarrow X(\omega)$$

$$\boxed{\int_{-\infty}^{\tau} x(\tau)\,d\tau \longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)}$$

Proof – $FT[x(t) * u(t)]$

$$= FT\{x(t)\} \times FT\{u(t)\}$$

$$= X(\omega)\cdot\left(\frac{1}{j\omega} + \pi\delta\omega\right)$$

$$= \frac{X(\omega)}{j\omega} + \pi X(\omega)\cdot\delta(\omega)$$

$$\because X(t_0)\cdot\delta(t-t_0) = X(t_0)\cdot\delta(t)$$

$$= \frac{X(\omega)}{j\omega} + \pi X(0)\cdot\delta(\omega)$$

(13) Differentiation in frequency :-

$$x(t) \longleftrightarrow X(\omega)$$

$$\boxed{t^n\cdot x(t) \longleftrightarrow (j)^n \frac{d^n}{d\omega^n}[X(\omega)]}$$

(14) Modulation :-

$$x(t) \rightleftharpoons X(j\omega)$$

(i) $x(t)\cdot \cos\omega_0 t \longleftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$

(ii) $x(t)\sin\omega_0 t \longleftrightarrow \frac{j}{2}[X(\omega+\omega_0) - X(\omega-\omega_0)]$

(15) Duality :-

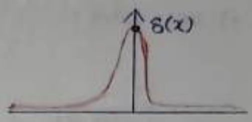
$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi x(-\omega)$$

* Unit Impulse function :- (Delta fn)

↳ This is a Narrow spike with area 1.

$$\delta(x) = \begin{cases} 0, & x \neq 0 \\ \infty, & x = 0 \end{cases}$$




$$\int_{-\infty}^{\infty} \delta(x)\,dx = 1$$

- $f(x)\cdot\delta(x) = f(0)\,\delta(x)$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\cdot\delta(x) = f(0)$$

- $f(x)\cdot\delta(x-a) = f(a)\cdot\delta(x-a)$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\cdot\delta(x-a) = f(a)$$

- $x(t)\cdot\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$
- $x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$
- $\delta(at) = \frac{1}{|a|}\delta(t)$
- $\delta(-t) = \delta(t)$

## * FT of Basic Signals :-

1) DC - Value :-

$$x(t) = A_0$$

$$\Rightarrow \int_{-\infty}^{\infty} x(t)\cdot dt = \int_{-\infty}^{\infty} A_0\, dt = \infty$$

(Not Integrable)

$$x(t) \longleftrightarrow X(\omega) = A_0\delta(\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\cdot e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A_0\,\delta(\omega)\cdot e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} A_0 \int_{-\infty}^{\infty} \delta(\omega-0)\cdot e^{j\omega t}\, d\omega$$

$$= \frac{A_0}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, d\omega$$

$$x(t) = A_0/2\pi \longleftrightarrow A_0\delta(\omega)$$

$$A_0 \longleftrightarrow 2\pi A_0\delta(\omega)$$

2) Impulse signal :-

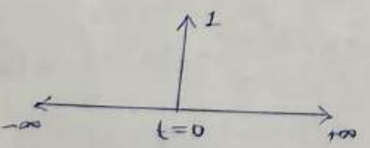
$$x(t) = \delta(t)$$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t-0)\cdot e^{-j\omega t}\, dt$$

$$\boxed{X(\omega) = 1}$$




(3) Exponential signals :-

$$x(t) = e^{-at}u(t), \quad a>0$$

$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \cancel{x(t)}\, e^{-at}\cdot u(t)\cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\cdot 1\cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{-1}{a+j\omega}\left[e^{-(a+j\omega)}\right]_0^{\infty}$$

$$\boxed{X(\omega) = \frac{1}{a+j\omega}}$$

(4) Signum function :-

$$x(t) = sgn(t) = \begin{cases} -1, & t<0 \\ 0, & t=0 \\ 1, & t>1 \end{cases}$$

$$x(t) \longleftrightarrow X(\omega)$$

$\because x(t)$ is Not absolute Integrable.

So Consider- $x(t) = \lim_{a\to 0}\left[e^{-a|t|} sgn(t)\right]$

$$\because sgn(t) = u(t) - u(-t)$$

$$\Rightarrow x(t) = \lim_{a\to 0}\left[e^{-at}u(t) - e^{at}u(-t)\right]$$

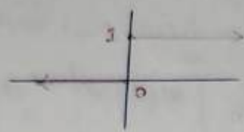
$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty}\left(\lim_{a\to 0}\left[e^{-at}u(t) - e^{at}u(-t)\right]e^{-j\omega t}\right.$$

$$= \lim_{a\to 0}\left[\int_{0}^{\infty} e^{-(a+j\omega)t}\, dt - \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt\right]$$

$$\boxed{X(\omega) = \frac{2}{j\omega}}$$

* **Unit step fn :-** $u(t)$

$$u(t) = \begin{cases} 1 , & t > 0 \\ 0 , & t < 0 \end{cases}$$




→ Property :-

(i) $[u(t)]^n = u(t)$

(ii) $[u(t-t_0)]^k = u(t-t_0)$

(iii) $u(at) = u(t)$

(iv) $u(at-t_0) = u(t - t_0/a)$

* **Unit Ramp Signal :-** $r(t)$

$$r(t) = \begin{cases} t , & t \geq 0 \\ 0 , & t < 0 \end{cases}$$

$$\Rightarrow r(t) = \int u(t)\, dt$$

* **Signum fn :-** $sgn(t)$

$$sgn(t) = \begin{cases} +1 , & t > 0 \\ 0 , & t = 0 \\ -1 , & t < 0 \end{cases}$$

$$\Rightarrow sgn(t) = 2u(t) - 1$$
$$= u(t) - u(-t)$$

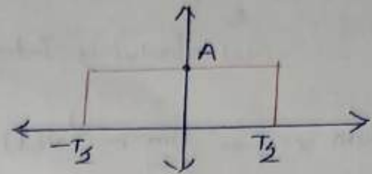
* **Unit Parabolic Signal :-**

$$x(t) = \begin{cases} t^2/2 , & t \geq 0 \\ 0 , & t < 0 \end{cases}$$

$$x(t) = \int r(t)\, dt = \iiint \delta(t)\, dt$$

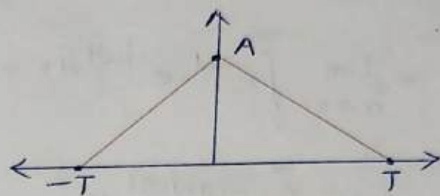
* **Rectangular pulse -**

$$x(t) = A \, rect\left(\frac{t}{T}\right)$$




* **Tringular signal :-**

$$x(t) = A\left(1 - \frac{|t|}{T}\right)$$




* **Sinusodial fn :-**

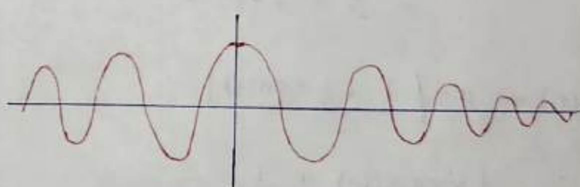
$$x(t) = A \cos(\omega_0 t + \phi)$$
$$= A \sin(\omega_0 t + \phi)$$
$$\omega_0 = 2\pi f , \quad T = \frac{1}{f} = \frac{2\pi}{\omega_0}$$

* **Sinc fn :-**

↳ Normalized fn

↳ $Sinc(\lambda) = \dfrac{\sin(\pi\lambda)}{\pi\lambda}$

$= 0$ for $\lambda = \pm 1, \pm 2 \ldots$



* **Sampling fn :-**

↳ un-Normalized fn

↳ $Sa(\lambda) = \dfrac{\sin\lambda}{\lambda}$

$= 0$ for $\lambda = \pm\pi, \pm 2\pi \ldots$

$$Sinc(0) = Sa(0) = 1$$

(5) Unit step signal :-

$$x(t) = u(t)$$

$\Downarrow$

Not absolutely Integrable

Consider - $x(t) = \lim_{a\to 0} e^{-at}u(t)$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} \lim_{a\to 0} e^{-at}u(t)\,dt$$

$$= \lim_{a\to 0} \int_{0}^{\infty} e^{-at}\cdot e^{-j\omega t}\,dt$$

$$= \lim_{a\to 0} \int_{0}^{\infty} e^{-(a+j\omega)t}\,dt$$

$$= \lim_{a\to 0} \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$= \lim_{a\to 0} \left[\frac{0-1}{-(a+j\omega)}\right]$$

$$= \lim_{a\to 0} \frac{1}{a+j\omega} = \frac{1}{j\omega}$$

Consider - $x(t) = \frac{1+\mathrm{sgn}(t)}{2}$

$$= \frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t)$$

$$FT(x(t)) = FT\left[\frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t)\right]$$

$$\frac{1}{2}\times 2\pi\,\delta(\omega) + \frac{1}{2}\cdot\frac{2}{j\omega}$$

$$\boxed{X(\omega) = \frac{1}{j\omega} + \pi\,\delta(\omega)}$$

(6) FT of $e^{j\omega_0 t}$ :-

$$x(t) = e^{j\omega_0 t}$$

$$x(t) \longleftrightarrow X(\omega)$$

$$X'(\omega) = \delta(\omega-\omega_0) \rightleftharpoons x'(t)$$

$$x'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X'(\omega)\,e^{j\omega t}\,d\omega$$

$$x'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\,e^{j\omega t}\,d\omega$$

$$x'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\cdot e^{j\omega_0 t}\,d\omega$$

$$= \frac{e^{j\omega_0 t}}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\,d\omega$$

$$= \frac{1}{2\pi}e^{j\omega_0 t}$$

$$\frac{e^{j\omega_0 t}}{2\pi} \longleftrightarrow \delta(\omega-\omega_0)$$

$$\boxed{e^{j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega-\omega_0)}$$

(7) FT of $\cos\omega_0 t$ :-

$$x(t) = \cos\omega_0 t$$

$$x(t) = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$FT[x(t)] = \frac{1}{2}\left[FT(e^{j\omega_0 t}) + FT(e^{-j\omega_0 t})\right]$$

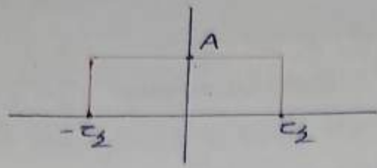
$$X(\omega) = \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_0) + 2\pi\,\delta(\omega+\omega_0)\right]$$

$$\boxed{X(\omega) = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]}$$

$$\because \sin\omega_0 t = \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right]$$

$$\sin\omega_0 t \longleftrightarrow \pi j\left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right]$$

(8) Rectangular fn :-




$$x(t) = \begin{cases} A, & -\tau/2 \le t \le \tau/2 \\ 0, & \text{else} \end{cases}$$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(\omega) = \int_{-\tau/2}^{\tau/2} A\cdot e^{-j\omega t}\,dt$$

$$X(\omega) = \frac{A}{(-j\omega)}\left[e^{-j\omega t}\right]_{-\tau/2}^{\tau/2}$$

$$X(\omega) = \frac{-A}{j\omega}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right]$$

$$= \frac{A}{j\omega}\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

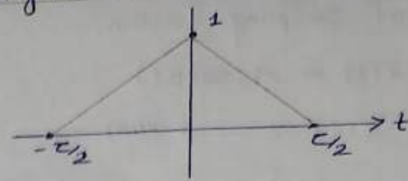
$$= \frac{2A}{\omega}\left[\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right]$$

$$X(\omega) = \frac{2A}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega) = A\tau\cdot\frac{\sin(\omega\tau/2)}{\omega\tau/2}$$

$$\boxed{X(\omega) = A\tau\cdot \text{sinc}\left(\frac{\omega\tau}{2}\right)}$$

(9) Tringular pulse :-




$$\Delta\left(\frac{t}{\tau}\right) = 1 - \frac{2|t|}{\tau}, \quad |t| < \frac{\tau}{2}$$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} \Delta\left(\frac{t}{\tau}\right) e^{-j\omega t}\,dt$$

$$= \int_{-\tau/2}^{0}\left(1+\frac{2t}{\tau}\right)e^{-j\omega t}dt + \int_{0}^{\tau/2}\left(1-\frac{2t}{\tau}\right)e^{-j\omega t}$$

$$= \int_{0}^{\tau/2}\left(1-\frac{2t}{\tau}\right)e^{j\omega t}dt + \int_{0}^{\tau/2}\left(1-\frac{2t}{\tau}\right)e^{-j\omega t}$$

$$= \int_{0}^{\tau/2}\left[e^{j\omega t}+e^{-j\omega t}\right]dt - \frac{2}{\tau}\int_{0}^{\tau/2} t\cdot\left[e^{j\omega t}+e^{-j\omega t}\right]dt$$

$$= \int_{0}^{\tau/2} 2\cos\omega t\,dt - \frac{2}{\tau}\int_{0}^{\tau/2} 2t\cos\omega t\,dt$$

$$X(\omega) = \frac{4}{\omega^2\tau}\left[1-\cos\frac{\omega\tau}{2}\right]$$

$$X(\omega) = \frac{4}{\omega^2\tau}\cdot 2\sin^2\left(\frac{\omega\tau}{4}\right)$$

$$X(\omega) = \frac{8}{\omega^2\tau}\left[\frac{\sin^2(\omega\tau/4)}{\left(\frac{\omega\tau}{4}\right)^2}\right]\cdot\left(\frac{\omega\tau}{4}\right)^2$$

$$\boxed{X(\omega) = \frac{\tau}{2}\,\text{sinc}^2\left(\frac{\omega\tau}{4}\right)}$$

10) FT of Sampling function:-

$$x(t) = A_0 Sa(Kt)$$

$$x(t) \longleftrightarrow X(\omega)$$

$\because$ $y(t) = A\,Rect\left(\frac{t}{\tau}\right) \longleftrightarrow A\tau Sa\left(\frac{\omega\tau}{2}\right)$

by Duality property-

$$\omega = t$$

$$t = -\omega$$

$$A\tau Sa\left(\frac{\omega t}{2}\right) \longleftrightarrow 2\pi y(-\omega)$$

$$A_0 = A\tau$$

$$K = \omega/2$$

## * Amplitude modulation :-

→ change in the amplitude of the carrier signal with respect to the instantaneous value of the amplitude of the baseband message signal.

$$[x_c(t)]_{Am} = m(t)\cdot \cos\omega_c t + A_c \cos\omega_c t$$

$$\because FT(\cos\omega_c t) = \pi(\delta(\omega-\omega_c)+\delta(\omega+\omega_c))$$

$$FT[A_c\cos\omega_c t] = A_c\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$$

$$FT[m(t)\cdot\cos\omega_c t] = \frac{1}{2}[M(\omega-\omega_c)+M(\omega+\omega_c)]$$

$$\boxed{[X_c(\omega)]_{Am} = A_c\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \frac{1}{2}[M(\omega-\omega_c)+M(\omega+\omega_c)]}$$

• Single tone modulation :-

$$\because [x_c(t)]_{Am} = (A_m\cos\omega_m t + A_c)\cos\omega_c t$$

$$= A_c\left[1+\frac{A_m}{A_c}\cos\omega_m t\right]\cdot\cos\omega_c t$$

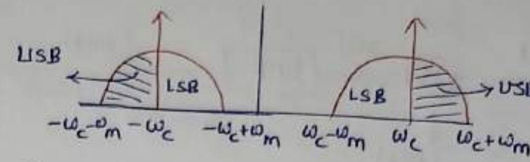
$$\text{let } \mu = \frac{A_m}{A_c}$$

$$= (A_c\cdot\cos\omega_c t)(1+\mu\cos\omega_m t)$$

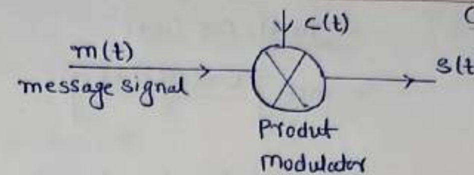
$$= A_c\cos\omega_c t + \mu A_c\cos\omega_m t\cos\omega_c t$$

$$= A_c\cos\omega_c t + \frac{\mu A_c}{2}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$


→ Spectrum of $[X_c(\omega)]_{Am}$




(1) DSB-SC (Double Side Band - Suppressed Carrier) :-




$$s(t) = m(t)\cdot c(t)$$

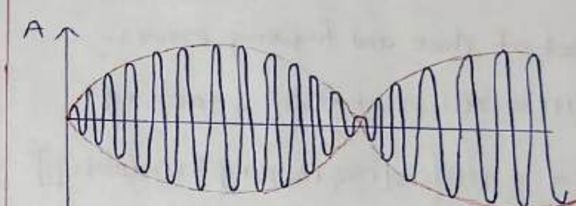
$$s(t) = m(t)\cdot A_c\cos\omega_c t$$

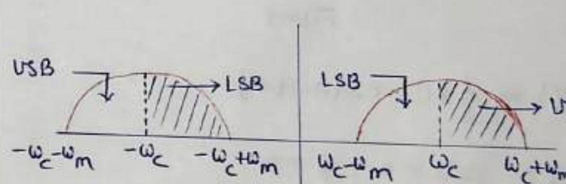
$$m(t) \longleftrightarrow M(\omega)$$

$$s(t) = \frac{1}{2}A_c m(t)[e^{j\omega_c t}+e^{-j\omega_c t}]$$

$$= \frac{1}{2}A_c[m(t)e^{j\omega_c t} + m(t)e^{-j\omega_c t}]$$

$$s(t) \longleftrightarrow \frac{1}{2}A_c[M(\omega_c-\omega_m)+M(\omega_c+\omega_m)]$$




* Power Required to transmit one side band -

$$P_{\text{one side band}} = \frac{1}{8}\mu^2 A_c^2$$

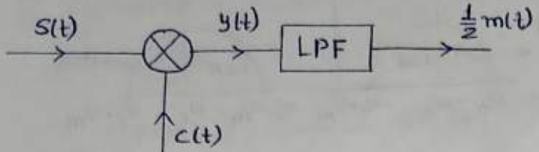
$$P_{\text{both side band}} = \frac{1}{4}\mu^2 A_c^2$$

$$P_{\text{Carrier}} = \frac{1}{2}A_c^2$$

* Band - Width = $BW_{DSB\text{-}SC}$

$$= (\omega_c+\omega_m)-(\omega_c-\omega_m) = 2\omega_m$$

## * Synchronous De-modulator:-



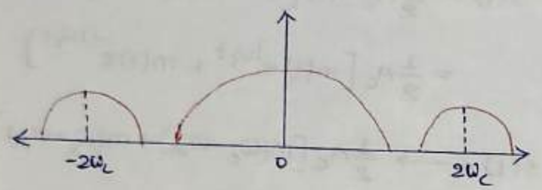

$$y(t) = s(t) \cdot c(t)$$

$$= m(t) \cdot \cos^2(\omega_c t)$$

$$= m(t) \left[\frac{\cos(2\omega_c t)+1}{2}\right]$$

$$= \underbrace{\frac{1}{2} m(t) \cdot \cos(2\omega_c t)}_{\text{Rejected by LPF}} + \frac{1}{2} m(t)$$

$$Y'(\omega) = \frac{1}{2} M(\omega) + \frac{1}{4}\left[M(\omega - 2\omega_c) + M(\omega + 2\omega_c)\right]$$




## * Effect of Phase and frequency Error :-

$$e_d(t) = x(t) \cos\omega_c t \cdot \cos[(\omega_c + \Delta\omega)t + \phi]$$

$$e_d(t) = \frac{1}{2} x(t) \{\underbrace{\cos[(2\omega_c + \Delta\omega)t + \phi]}_{\text{Filter}} + \cos(\Delta\omega t + \phi)\}$$

$$e_d(t) = \frac{1}{2} x(t) \cdot \cos(\Delta\omega t + \phi)$$

Case-1 : $\Delta\omega = 0, \ \phi = 0$

$$e_d(t) = \frac{1}{2} x(t) \rightarrow \text{No Error}$$

Case-2 : $\Delta\omega = 0, \ \phi \neq 0$

$$e_d(t) = \frac{1}{2} x(t) \cos\phi$$

→ Attenuation Error (signal strength ↓)

When – $\phi = 90°$

$$e_d(t) = 0$$

↳ Quadrature Null Effect

Case-3 : $\Delta\omega \neq 0, \ \phi = 0$

$$e_d(t) = \frac{1}{2} x(t) \cos(\Delta\omega t)$$

→ distortion Error (quality ↓)

## * Power Efficiency –

$$\because m(t) = A_m \cos(\omega_m t)$$

$$c(t) = A_c \cos(\omega_c t)$$

$$s(t) = \frac{A_c A_m}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

$$P_{SB} = \frac{1}{2}\left(\frac{A_c A_m}{2}\right)^2 + \frac{1}{2}\left(\frac{A_c A_m}{2}\right)^2$$

$$\boxed{P_{SB} = \frac{1}{4} A_c^2 A_m^2}$$

$$P_T = P_{SB}$$

$$\boxed{\eta = \frac{P_{LSB} + P_{USB}}{P_T}}$$

## * Co-star Receiver :-

↳ It Removes the Quadrature Null Effect of ~~G~~ Synchronous demodulator.

Inphase Line –

$$y_I(t) = \frac{A_c^2}{2} m(t) \cos\phi$$

Q phase Line –

$$y_Q(t) = s(t) \cdot A_c \sin(\omega_c t + \phi)$$

$$= \frac{A_c^2}{2} m(t) \sin\phi$$

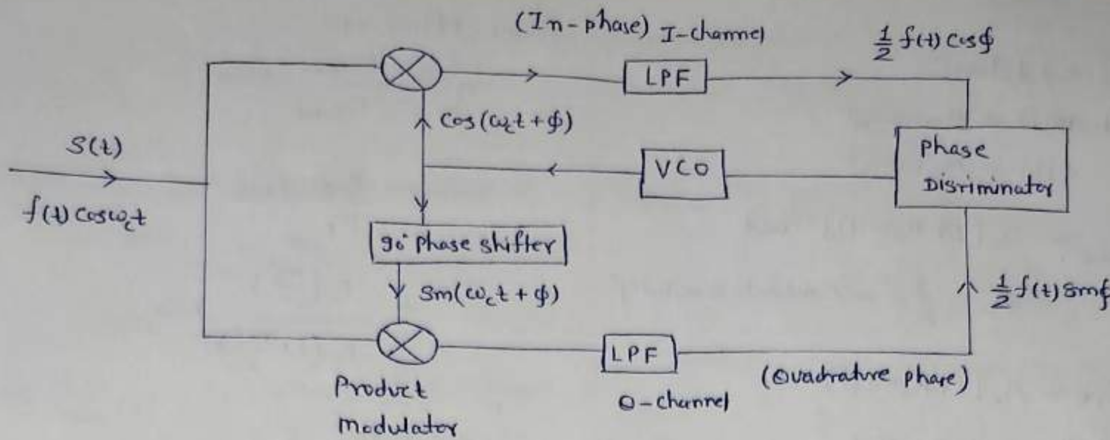
$$y(t) = y_I(t) + y_Q(t)$$

$$= \frac{A_c^2}{2} m(t) [\sin\phi + \cos\phi]$$

When $\phi = 90° \Rightarrow y_I(t) = 0$

$$y_Q(t) = \frac{A_c^2}{2} m(t)$$

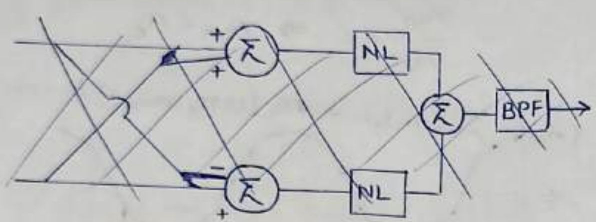
$$y(t) - \frac{1}{2} A_c^2 m(t)$$

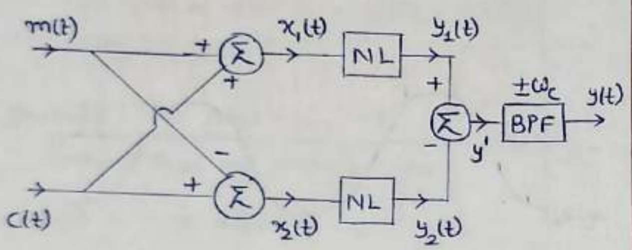



* Co-Star Reciever *

Note → detector – "Pielet Receiver"

* Non - Linear modulator :-




$$x_1(t) = m(t) + A_c \cos\omega_c t$$

$$x_2(t) = A_c \cos\omega_c t - m(t)$$

$$\because y(t) = a x(t) + b x^2(t) + \ldots\ldots$$

$$y' = y_1(t) - y_2(t)$$

$$= [a x_1(t) + b x_1^2(t)] - [a x_2(t) + b x_2^2(t)]$$

$$= [a(m(t) + A_c \cos\omega_c t) + b(m(t) + A_c \cos\omega_c t)^2] - [a(A_c \cos\omega_c t - m(t)) + b(A_c \cos\omega_c t - m(t))^2]$$

$$= 2a\, m(t) + 4b\, m(t) \cos\omega_c t \cdot A_c$$

$$y' = 2a\, m(t) + 4A_c b\, m(t) \cos\omega_c t$$

$$\boxed{y(t) = 4A_c b\, m(t) \cdot \cos\omega_c t}$$

## * AM-modulation:-

Let $m(t) = A_m \cos\omega_m t$

$c(t) = A_c \cos\omega_c t$

$$[s(t)]_{AM} = A_c[1+K_a m(t)]\cos\omega_c t$$

$K_a = \frac{1}{A_c}$ = "Amplitude sensitivity"

$$\text{Envelope} = A_c[1+K_a m(t)]$$

→ Called it as "DSB-FC"

· Single Tone AM :-
↳ single message signal

$m(t) = A_m \cos\omega_m t$

$$[s(t)]_{AM} = A_c[1+K_a m(t)]\cos\omega_c t$$

$$= A_c[1+K_a A_m \cos\omega_m t]\cos\omega_c t$$

$$= A_c[1+m_a \cos\omega_m t]\cos\omega_c t$$

$$\begin{cases} m_a = K_a A_m \text{ (Modulation Index)} \\ m_a = \frac{A_m}{A_c} \end{cases}$$

$$= A_c\cos\omega_c t + A_c m_a \cos\omega_m t \cos\omega_c t$$

$$= A_c\cos\omega_c t + \frac{A_c m_a}{2}[2\cos\omega_m t \cos\omega_c t]$$

$$= A_c\cos\omega_c t + \frac{A_c m_a}{2}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

⇒ Power-Analysis:-

$$P_c = \frac{1}{2}A_c^2$$

$$P_{USB} = \frac{1}{8}A_c^2 m_a^2 = \frac{1}{4}P_c m_a^2$$

$$P_{LSB} = \frac{1}{8}A_c^2 m_a^2$$

$$P_{SB} = P_{USB} + P_{LSB} = \frac{1}{4}A_c^2 m_a^2$$

$$P_T = \frac{1}{2}A_c^2\left[1+\frac{1}{2}m_a^2\right]$$

$$= P_c\left[1+\frac{m_a^2}{2}\right]$$

Power-Efficiency -

$$\eta = \frac{P_{useful}}{P_{Total}} \times 100$$

$$= \frac{P_{SB}}{P_T} \times 100$$

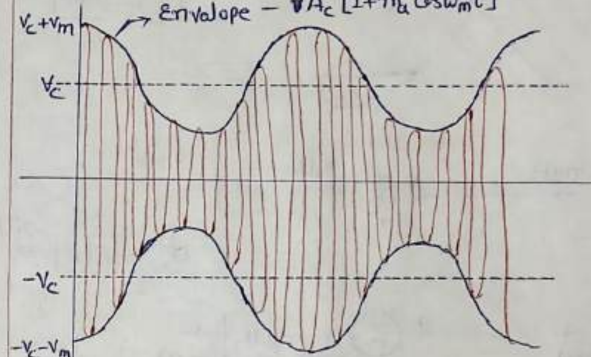
$$= \frac{P_c\left(\frac{m_a^2}{2}\right)}{P_c\left(1+\frac{m_a^2}{2}\right)} \times 100$$

$$\boxed{\eta = \left(\frac{m_a^2}{2+m_a^2}\right) \times 100}$$

$m_a = 1 \Rightarrow \eta = \frac{100}{3} = 33.33\%$

Case-1 : $m_a < 1 \Rightarrow$ under-modulation

$\frac{A_m}{A_c} < 1 \Rightarrow A_m < A_c$




$$|s(t)|_{max} = A_c + A_m$$

$$|s(t)|_{min} = A_c - A_m$$

$$A_c = \frac{[s(t)]_{max} + [s(t)]_{min}}{2}$$

$$A_c = \frac{A_{max} + A_{min}}{2}$$

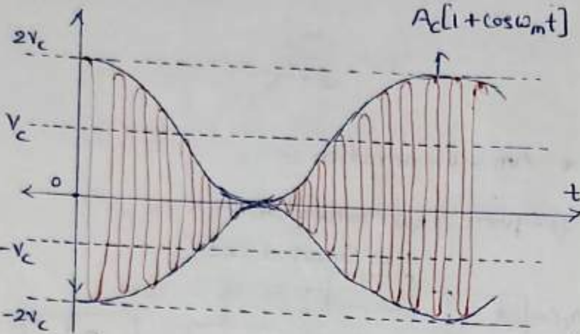
$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$\boxed{m_a = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

**Case-2:-** $m_a = 1$ (Critical modulation)

$\rightarrow [s(t)]_{max} = 2V_c$

$\rightarrow [s(t)]_{min} = 0$




**Case-3:-** $m_a > 1$ (over modulation)

$A_c < A_m$

→ due to the phase Reversal, data will be lost in this Case, so we do not acquire this.

*** multi-Tone Am:-**

↳ more than one message signal.

$m(t) = m_1(t) + m_2(t) + \ldots\ldots$

$s(t)\Big|_{Am} = A_c[1 + m_{a1}\cos\omega_{m_1}t + m_{a_2}\cos\omega_{m_2}t + \ldots]$

• Power-

$P_{USB} = P_{LSB} = \frac{1}{4} P_c [m_{a_1}^2 + m_{a_2}^2 + \ldots\ldots]$

$$\because \boxed{m_a = \sqrt{m_{a_1}^2 + m_{a_2}^2 + m_{a_3}^2 + \ldots}}$$

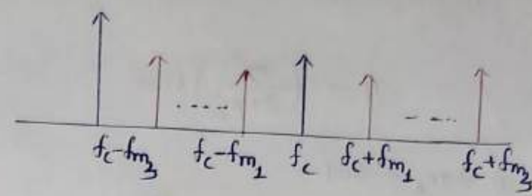
↓

Net modulation Index

$P_T = P_c\left(1 + \frac{m_a^2}{2}\right)$

• Band-width:-

Let $f_{m_1} < f_{m_2} < f_{m_3}$




$B.W = 2f_{m_3} = 2 \cdot \max\{f_{m_1}, f_{m_2}, f_{m_3} \ldots\}$

**Q.** for an Am Signal, BW is 10 KHZ and the highest frequency Component present is 705 KHZ. What is the Carrier freq. used for Am signal.

**Ans** $BW = 10KHZ$

$f_{max} = 705KHZ$

$\Rightarrow 2f_m = 10 \Rightarrow f_m = 5KHZ$

$\because f_c + f_m = 705$

$f_c = 700KHZ$

~~**Q.** for an Am signal, The BW is 20KHZ. The fi~~

A bradcaster Am transmitter used Carrier Power of 400W and depth of modulation is 75%. What is the total Power in modulated wave.

**Ans-** $P_c = 400W$, $\therefore m_a = 75\%$

$m_a = 3/4$

$P_T = P_c\left[1 + \frac{m_a^2}{2}\right]$

$= 400\left(1 + \frac{9}{32}\right) = \frac{41 \times 400}{32}$

$= 512.5W$

**Q.** find the efficiency of Am signal of -

(a) $\% m_a = 60\%$ (b) $\% m_a = 50\%$.

**Ans-** $\because \eta = \left(\frac{m_a^2}{2+m_a^2}\right) \times 100$

a) $\% m_a = 60\%$.

$\rightarrow m_a = 6/10 = 3/5$

$$\eta = \left[\frac{9/25}{2+9/25}\right] \times 100$$

$$\eta = \frac{9}{59} \times 100 = 15.25\%$$

**Q.** $\phi(t) = 10\cos(2\pi 10^6 t) + 5\cos(2\pi 10^6 t)\cos(2\pi 10^3 t) + 2\cos(2\pi 10^6 t)\cos(4\pi 10^3 t)$

draw the line spectrum, find modulation Index.

**Ans⇒** $\phi(t) = 10\left[1 + 0.5\cos(2\pi 10^3 t) + 0.2\cos(4\pi 10^3 t)\right]\cos(2\pi 10^6 t)$

$m_{a_1} = 0.5$ $\quad \omega_c = 2\pi 10^6$

$m_{a_2} = 0.2$ $\quad \omega_{m_1} = 2\pi \times 10^3$

$\omega_{m_2} = 4\pi \times 10^3$

USB: $f_c + f_{m_1}$, $f_c + f_{m_2}$

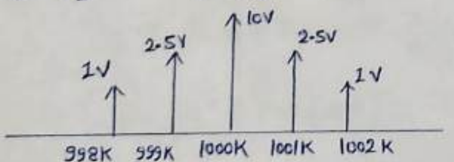
$= [10^6 + 10^3,\ 10^6 + 2\times 10^3]$

LSB: $[f_c - f_{m_1},\ f_c - f_{m_2}]$

$= [10^6 - 10^3,\ 10^6 - 2\times 10^3]$

$$A_{f_c+f_{m_1}} = A_{f_c-f_{m_1}} = \frac{A_c}{2} \cdot m_{a_1} = 2.5\text{V}$$

$$A_{f_c+f_{m_2}} = A_{f_c-f_{m_2}} = \frac{A_c}{2} m_{a_2} = 1\text{V}.$$




**Note-**

- Square wave form-

$$P_{Am} = P_c(1 + m_a^2)$$

$$\eta = \frac{m_a^2}{1+m_a^2}$$

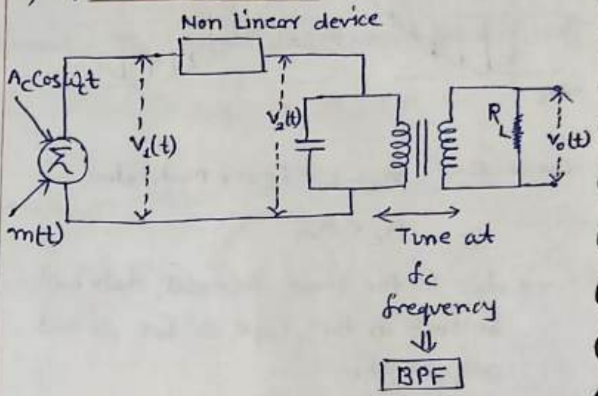
- Tringular wave form-

$$P_{Am} = P_c\left(1 + \frac{m_a^2}{3}\right)$$

$$\eta = \frac{m_a^2}{3+m_a^2}$$

*** Am - Generation :-**

1) Square Law modulation :-



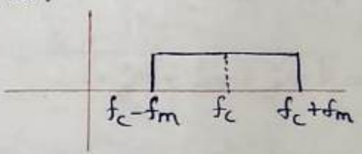

$\because V_1(t) = m(t) + A_c \cos\omega_c t$

$V_2(t) = aV_1(t) + bV_1^2(t)$

$V_2(t) = a(m(t) + A_c\cos\omega_c t) + b[m(t) + A_c\cos\omega_c t]^2$

$= am(t) + aA_c\cos\omega_c t + bm^2(t) + bA_c^2\cos^2\omega_c t + 2bm(t)A_c\cos\omega_c t$

$\because$ BPF-




This Can pass only in the Range of $(f_c - f_m,\ f_c + f_m)$

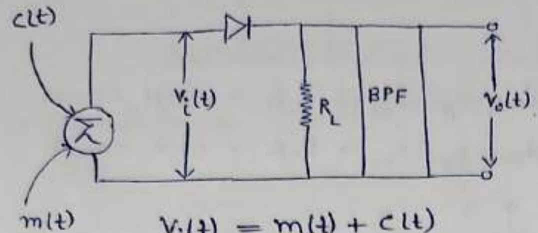
$y(t) = V_0(t) = aA_c\cos\omega_c t + 2bm(t)A_c\cos\omega_c t$

$= A_c[a + 2bm(t)]\cos\omega_c t$

$= aA_c\left[1 + \frac{2b}{a}m(t)\right]\cos\omega_c t$

$$m_a = \frac{2b}{a} A_m$$

*** Switching modulator :-**




$$V_i(t) = m(t) + c(t)$$

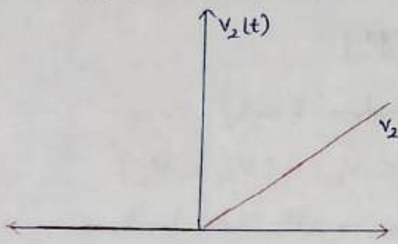
if $v_i(t) > 0 \Rightarrow$ forward bias $\Rightarrow$ Closed switch

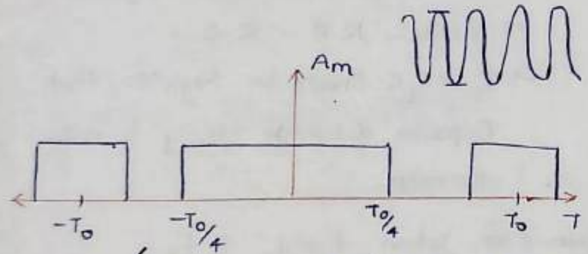
if $v_i(t) < 0 \Rightarrow$ Reverse bias $\Rightarrow$ open switch

$\because A_c >> A_m \rightarrow$ diode depends on $A_c$

$V_2(t) = V_i(t)$ if $c(t) > 0$

$V_2(t) = 0$ if $c(t) \leq 0$




↳ functioning of diode = P(t)

$$V_2(t) = V_i(t) \cdot P(t)$$

$$V_i(t) = m(t) + A_c \cos\omega_c t$$

$$P(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[\omega_c t(2n-1)]$$

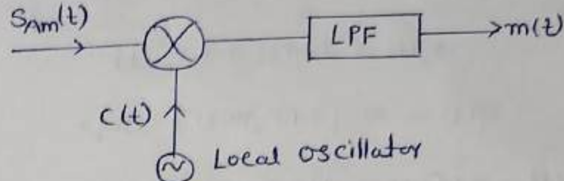
$$V_2(t) = V_i(t) \left[\frac{1}{2} + \frac{2}{\pi} \cdot \frac{1}{1} \cos\omega_c t + \text{odd harmonic } (f_c \uparrow)\right]$$

$$V_2(t) = (m(t) + A_c \cos\omega_c t)\left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_c t\right]$$

$$V_o(t) = \frac{A_c}{2}\cos\omega_c t + \frac{2}{\pi} m(t) \cos\omega_c t$$

$$\frac{A_c}{2}\left(1 + \frac{4}{\pi A_c} m(t)\right) \cos\omega_c t$$

*** Synchronous / coherent Detector :-**




$$S_{Am}(t) = A_c[1 + K_a m(t)] \cos\omega_c t$$

$$c(t) = A_c \cos\omega_c t$$

$$y(t) = S_{Am}(t) \cdot c(t)$$

$$= A_c^2 (1 + K_a m(t)) \cdot \cos^2(\omega_c t)$$

$$= A_c^2 [1 + K_a m(t)] \cdot \left[\frac{1 + \cos(2\omega_c t)}{2}\right]$$

$$= \underset{\times}{\frac{A_c^2}{2}} + \frac{A_c^2 K_a m(t)}{2} + \underset{\times}{A_c^2 (K_a m(t))[\;]}$$

$$y'(t) = \frac{A_c^2 K_a}{2} m(t)$$

\# If No phase synchronization

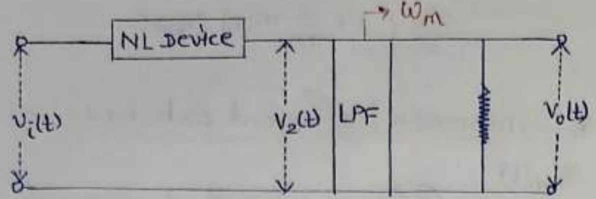
$$c(t) = A_c \cos(\omega_c t + \phi)$$

$$y'(t) = \frac{A_c^2 K_a}{2} m(t) \cdot \cos\phi$$

if $\phi = 90^\circ \Rightarrow y'(t) = 0$

(Quadrature Null Effect)

## * Square - Law Detection :-




$$V_2(t) = a v_i(t) + b v_i^2(t)$$

$$V_i(t) = A_c[1+K_a m(t)]\cos\omega_c t$$

$$V_2(t) = a\{[1+K_a m(t)]A_c\cos\omega_c t\} + b\{[1+K_a m(t)]A_c\cos\omega_c t\}^2$$

$$V_2(t) = \cancel{aA_c\cos\omega_c t} + \cancel{aK_a m(t)A_c\cos\omega_c t} + b(1+K_a^2 m^2(t)+2K_a m(t))\left[A_c^2\frac{1+\cos 2\omega_c t}{2}\right]$$

$$V_o(t) = \frac{bA_c^2}{2}[1+K_a^2 m^2(t)+2K_a m(t)]$$

$$V_o(t) = \underset{\substack{\uparrow \\ \text{original}}}{bA_c^2\cdot K_a m(t)} + \underset{\substack{\uparrow \\ \text{Distortion}}}{\frac{bA_c^2}{2}K_a^2 m^2(t)}$$

$$D = \frac{bA_c^2 K_a m(t)}{\frac{bA_c^2}{2}K_a^2 m^2(t)} = \frac{2}{K_a m(t)}$$

$$\boxed{D = \frac{2}{K_a m(t)}}$$

for $D\downarrow$ $m_a\uparrow$

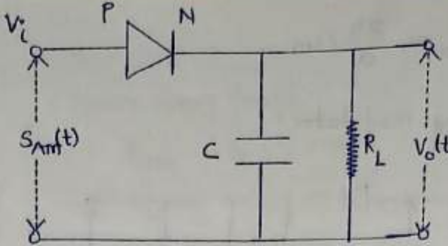
## * Envelope Detector :-

→ Simple and effective for detection of narrow band signal. ($f_c >> 2f_m$)

→ Used in Commercial AM radio receiver.

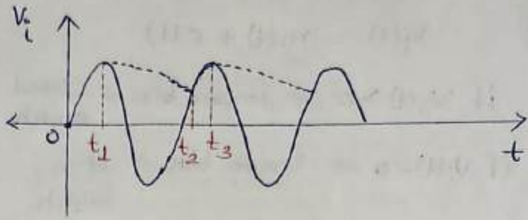
→ It can be used for - $\mu \le 1$

failed for - $\mu > 1$




When $V_P > V_N$ → F.B → short circuit

When $V_P < V_N$ → R.B → open circuit




Case-1 ⇒ when $t = 0^+$

$V_P > V_N$ → F.B → S.C.

→ Capacitor will charge

→ $\tau = RC$ should be small for rapid charging.

Case-2 ⇒ when $t = t_1^+$

$V_P < V_N$ ($V_{R_L} < V_C$)

↳ Capacitor will start discharge

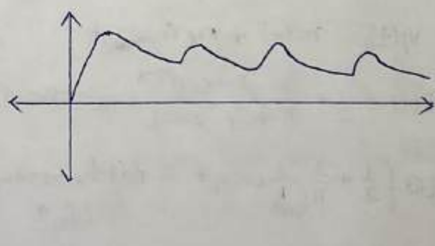
→ diode - R.B - O.C

→ $\tau = R_L C$ should be high so that Capacitor discharge slowly to follow Envelope.

Case-3 ⇒ when $t = t_2^+$ to $t_3$

$V_P > V_N$ → F.B → S.C

→ Capacitor will start charging.



If Input – $A_c[1+K_a m(t)]\cos\omega_c t$

Output – $A_c[1+K_a m(t)]$

$\Downarrow$ Passing through Capacitor

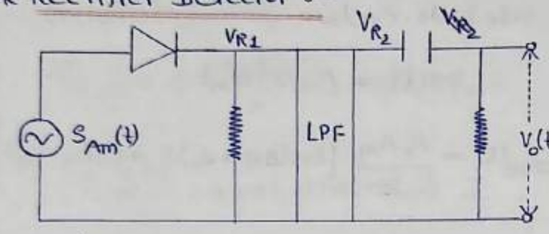
$A_c K_a m(t)$

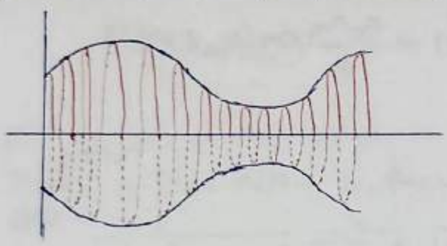
→ For Satisfactory opr. on Envelope detector – $\frac{1}{\omega_c} << RC << \frac{1}{\omega_B}$

$\downarrow$

Basement signal freq.

## * Rectifier Detector :-



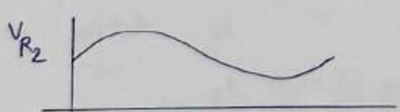
$$V_{R_1}(t) = \{[A+m(t)]\cos\omega_c t\}\cdot P(t)$$

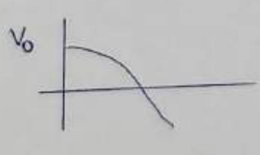
$$P(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t\right]$$

$$V_{R_2}(t) = [A+m(t)]\frac{1}{\pi}$$

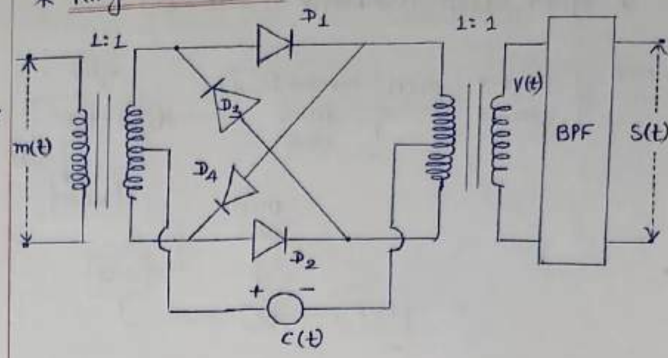
After passing LPF

$$V_{R_3}(t) = \frac{1}{\pi}m(t) = V_o(t)$$




## * Ring modulator :- (DSB-SC)




Case1 → when c(t) is (+)ive

$D_1$ & $D_2$ ⟶ Forward Bias and act as short circuit.

$D_3$ & $D_4$ ⟶ Reverse Bias and act as open circuit

$$V(t) = m(t)$$

Case-2 → when c(t) is (–)ive

$D_1$ & $D_2$ → Reverse Bias

$D_3$ & $D_4$ → forward Bias and act as short circuit.

$$V(t) = -m(t)$$

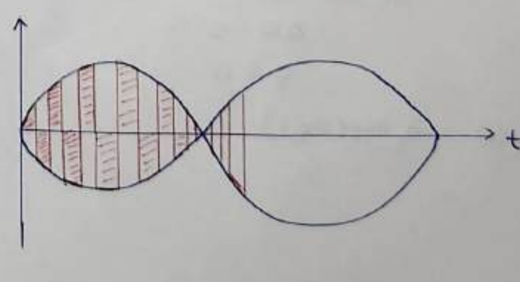
$$\Rightarrow V(t) = m(t)\cdot c(t)$$

Fourier series expansion of c(t) shows that it Contains lot of odd harmonics.

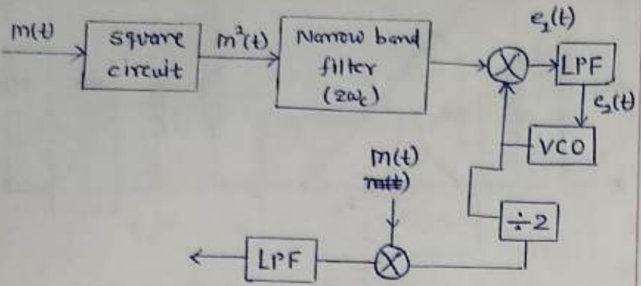
$$c(t) = \frac{4}{\pi}\sum_{n=1,3,5\ldots}\frac{(-1)^{n-\frac{1}{2}}}{n}\cos(n2\pi f_c t)$$

→ when V(t) is passed through BPF tuned to $f_c$

$$\boxed{S(t) = \frac{4}{\pi}m(t)\cos(2\pi f_c t)}$$



## * Square - Loop Technique (DSB-SC) :-




→ Demodulation Technique

→ $m(t) = A_c \cos\omega_c t \, m(t)$

$$m(t) = A_m \cos\omega_m t$$

→ $m^2(t) = (A_c^2 \cos^2\omega_c t)(A_m^2 \cos^2\omega_m t)$

$$= \frac{1}{4} A_c^2 A_m^2 (2\cos^2\omega_c t)(2\cos^2\omega_m t)$$

$$= \frac{1}{4} A_c^2 A_m^2 [1-\cos 2\omega_c t][1-\cos 2\omega_m t]$$

$$= A_K (1-\cos 2\omega_c t)(1-\cos 2\omega_m t)$$

$$= A_K [1-\cos 2\omega_c t - \cos 2\omega_m t + \cos 2\omega_c t \cos 2\omega_m t$$

After passing Narrow band filter-

$$= A_K \cos(2\omega_c t)$$

$$e_1(t) = A_K \cdot \cos(2\omega_c t)\{A_c \cos[(2\omega_c + \Delta\omega)t + \phi]\}$$

Let if vco generate -

$$A_c \cos[(2\omega_c + \Delta\omega)t + \phi]$$

$$= \frac{A_K A_c}{2} \cdot 2\cos[(2\omega_c+\Delta\omega)t+\phi] \cdot \cos(2\omega_c t)$$

$$= A_K' \{\cos[(4\omega_c+\Delta\omega)t+\phi] + \cos(\Delta\omega t+\phi)\}$$

$$e_2(t) = A_K' \cos(\Delta\omega t + \phi)$$

we set VCO -

$$\Delta\omega = 0$$

$$\phi = 0$$

$$c(t)\ \cancel{e_2(t)} = A_c \cos(2\omega_c t)$$

$$c'(t) = A_c \cos(\omega_c t)$$

$$s'(t) = m(t) \cdot c'(t)$$

$$s'(t) = (A_c \cos\omega_c t \cdot m(t)) \cdot A_c \cos\omega_c t$$

$$= \frac{A_c^2}{2} m(t)(2\cos^2\omega_c t)$$

$$= \frac{A_c^2 m(t)}{2}[1-\cos 2\omega_c t]$$

$$s(t) = \frac{A_c^2}{2} m(t)$$

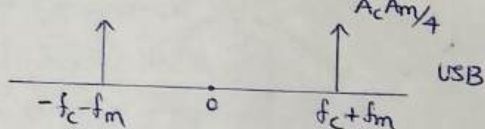
## * SSB (Single Side Band) :-

- In case of SSB, only one side band will be transmitted (because both the side bands contain same information)

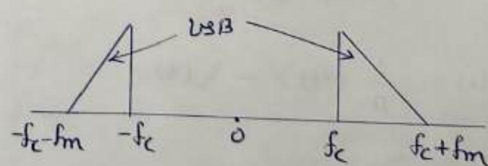
$$m(t) = A_m \cos\omega_m t$$

$$S_{DSB}(t) = \frac{A_c A_m}{2}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

$$\boxed{S_{SSB}(t) = \frac{A_c A_m}{2}\cos(\omega_c \pm \omega_m)t}$$




or


$$B.W = (f_c+f_m) - f_c$$

$$= f_m$$

- Power of SSB-

$$P_t = P_{USB} = P_{LSB}$$

$$P_t = \frac{1}{2}\left(\frac{A_c A_m}{2}\right)^2$$

$$\boxed{P_t = \frac{1}{8} A_c^2 A_m^2}$$

- Efficiency :-

$$\eta = \frac{P_{SB}}{P_t} = 1$$

$$\eta = 100\,\%$$

- General Expression :-

$$S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c \pm f_m) t$$

$$= \frac{A_c A_m}{2} \cos(2\pi f_c t)\cos(2\pi f_m t) \mp$$

$$\frac{A_c A_m}{2} \sin(2\pi f_c t) \sin(2\pi f_m t) \quad \begin{cases} - : USB \\ + : LSB \end{cases}$$

$\because m(t) = A_m \cos 2\pi f_m t$

↳ After 90° phase shift- $A_m \sin 2\pi f_m t$

$$= \hat{m}(t)$$

↳ $\hat{m}(t)$ is the hilbert transformation of $m(t)$.

$$\boxed{S_{SSB}(t) = \frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t}$$

$(-)$ : USB , $(+)$ : LSB